Augmented Information Rigidity Test*

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Abstract

The information rigidity test examines predictability of forecast errors from forecast revisions. This paper provides an augmented information rigidity test that controls for cross-sectional and time series correlations in panel data. Our Monte Carlo simulations show that the proposed tests have satisfactory size and power properties in typical macro panels. Applications of the tests to the U.S. Survey of Professional Forecasters covering 1968Q4 through 2016Q4 clearly reveal experts' overreaction to news in their macroeconomic expectations.

Keywords: Forecast evaluation, information rigidity, overreaction, panel data, Survey of Professional Forecasters

JEL Codes: C23, C53, E37

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1 Introduction

Full-information rational expectations (FIRE) require that forecast errors be unpredictable from forecast revisions. The seminal work of Coibion and Gorodnichenko (2015, CG hereafter) suggests testing this relationship by first aggregating the forecasts across individuals and then estimating the aggregated time series regression. In contrast, Bordalo, Gennaioli, Ma, and Shleifer (2020, BGMS hereafter) recommend estimating the average relationship by first running separate regressions for each individual and then aggregating (i.e. taking the mean or median of the coefficients obtained from the first step). While CG (2015) document the underreaction of consensus forecasts to news relative to FIRE, BGMS (2020) find that individual forecasters typically overreact. So there remains a puzzle about whether forecasters overreact or underreact. Given that the expectations formation process significantly affects macroeconomic dynamics and policy decisions (Mankiw and Reis, 2002; Sims, 2003; Woodford, 2003), a careful examination of the magnitude of deviation from FIRE cannot be overemphasized.

In this paper, we provide specific conditions under which both the CG (2015)'s "average then estimate" approach and the BGMS (2020)'s "estimate then average" approach are unbiased. The CG's approach yields unbiased estimates when the underlying error terms are cross-sectionally independent or when forecast revisions are cross-sectionally independent. The BGMS' approach gives unbiased estimates under cross-sectional independence of the error term in the regression of individual forecast errors on individual forecast revisions. In more general settings, however, both approaches suffer from severe biases. We borrow from and build on the work of Pesaran (2006) to propose an augmented information rigidity test that allows for cross-sectional and time series correlation. The idea is to augment the regression of individual forecast errors on individual forecast revisions by their cross-sectional averages.

The proposed estimators, labelled as common correlated effects (CCE) mean group (by allowing for slope heterogeneity) or pooled (by assuming slope homogeneity), have the correct size, satisfactory power for moderately large N and T, and very small root mean squared errors, which are comparable to those of the infeasible estimators. We apply the CCE-type estimators to test for rationality in the Survey of Professional Forecasters, and find overwhelming evidence that individual experts overreact to news in their macroeconomic expectations.

Our work contributes to the panel data literature on aggregation and pooling. See, for example, Baltagi, et al. (2000), Pesaran and Zhou (2018) and Wang, et al. (2019) for general discussions on "to pool or not to pool." Bonham and Cohen (2001) further argue that, due to heterogeneous individual forecasts and private-information bias, neither aggregation nor pooling is a valid strategy in testing the rational expectations hypothesis using survey data. We add to this literature by showing that the modified BGMS approaches of "estimate then average" and "pool" are appropriate by accounting for cross-sectional and time series correlations in a panel data setting.

Our work is also closely related to the recent macroeconomics literature in information rigidity. CG (2015), Drager and Lamla (2017) and Giacomini, et al. (2020) establish the presence of underreaction to news in inflation forecasts from professionals, consumers, and market participants. On the other hand, Bürgi (2016) and Crowe (2010) illustrate that the rigidity found at the aggregate level likely stems from the aggregation process. Messina, et al. (2015) show that the Greenbook forecasts made by the Federal Reserve staff are over-responsive to new information. Binder (2017) finds that both the low frequency and the rounding in household surveys result in overestimation of information stickiness. By matching a large database of individual macro forecaster data with the universe of sizable natural disasters across 54 countries, Baker, et al. (2020) document that information rigidity declines significantly following large shocks. BGMS (2020) present supporting evidence for the lack of information stickiness at the individual forecaster level. We add to this literature by proposing an augmented information rigidity test and documenting pervasive overreaction to news in professional's forecasts.

The paper proceeds as follows. Section 2 presents the information rigidity test and different estimators. Section 3 explores the small sample performance of various estimators in Monte Carlo experiments. Section 4 illustrates an empirical study in testing rationality in professional's forecasts. Section 5 concludes. Additional simulation results and derivations are relegated to the Appendices.

2 Models and Estimators

In this paper we consider the panel regression

$$y_{it} = \alpha_i + \beta_i \, x_{it} + e_{it}, \ i = 1, \ldots, N; \, t = 1, \ldots, T. \tag{1}$$

In the context of information rigidity testing, y_{it} is agent i's forecast error at time t, defined as the difference between actual value and forecast, and x_{it} is the forecast revision. Under FIRE, agent i's ex post forecast error is unpredictable from ex ante forecast revision. If this correlation is positive, upward revisions predict higher realizations compared to the forecasts, implying that forecasters underreact to information relative to FIRE. On the other hand, a negative correlation indicates overreaction. The forecast efficiency test in equation (1) was first proposed in Nordhaus (1987).

The error term in equation (1), e_{it}, is correlated across individuals and over time. The cross-sectional correlation in e_{it} is driven by the common factor f_t:

$$e_{it} = \gamma_i \, f_t + \xi_{it}, \tag{2}$$

and the factor loading γ_i measures the strength of error correlation. We allow for time series correlations in both the common factor f_t and in the idiosyncratic errors ξ_{it}:

$$f_t = \rho \, f_{t-1} + v_t, \tag{3}$$

and

$$\xi_{it} = \lambda_i \, \xi_{i,t-1} + u_{it}. \tag{4}$$

Furthermore, we assume that the regressor x_{it} in equation (1) is also subject to both cross-sectional and time series correlations, as is quite common in practice. Suppose that

$$x_{it} = \tau_i \, f_t + \epsilon_{it}, \tag{5}$$

and

$$\epsilon_{it} = \theta_i \, \epsilon_{i,t-1} + \eta_{it}. \tag{6}$$

Remark 1. Our model specification follows from Pesaran (2006), and nests other specifications used in the literature. For example, Driscoll and Kraay (1998) adopt a model including equations (1)-(4) by assuming a homogeneous slope coefficient (that is, $\beta_i = \beta$) and cross-sectional independence of x_{it}. Gow, et al. (2010), on the other hand, specify the model including equations (1)-(6) under two assumptions: (i) the slope coefficients are the same, i.e. $\beta_i = \beta$, and (ii) the factor loadings are the same, i.e. $\gamma_i = \gamma$ in equation (2) and $\tau_i = \tau$ in equation (5).

As a consequence of (2) and (5), we can rewrite (1) as

$$y_{it} = \alpha_i + (\beta_i \tau_i + \gamma_i) \, f_t + \beta_i \, \epsilon_{it} + \xi_{it}, \; i = 1, \ldots, N; \; t = 1, \ldots, T.$$

This makes explicit the role of the hidden factor f_t.

Consider further the homogeneous case, where $\beta_i = \beta$ is common. Then if we aggregate over the panels, we obtain

$$y_{\cdot t} = \sum_{i=1}^{N} \alpha_i + \beta \, x_{\cdot t} + \sum_{i=1}^{N} \gamma_i \, f_t + \xi_{\cdot t}. \tag{7}$$

So if we regress $y_{\cdot t}$ on $x_{\cdot t}$ (and a constant), there is a bias due to the presence of f_t. However, if $\sum_{i=1}^{N} \gamma_i = 0$ there is no bias. In such a case, the "aggregate" regression gives an unbiased estimate of β. In contrast, an Ordinary Least Squares (OLS) estimate based on pooling would regress y_{it} on a panel indicator and on x_{it}; in this case we have omitted the presence of $\gamma_i \, f_t$, which generates bias. We will explore the asymptotic bias at the end of this section and the small-sample bias in the next section.

Since both aggregate OLS and pooled OLS can potentially generate biased estimates, it is important to consider alternative estimators that appropriately handle the presence of common factors. In this paper we consider the method of Pesaran (2006) to obtain the common correlated effects estimators, which are appropriate for handling both heterogeneous slope ($\beta_i \neq \beta$) and a homogeneous slope ($\beta_i = \beta$). These estimators are defined explicitly below.

We first set out the general notation, following Pesaran (2006): let $\mathbf{d}_t$ be the M-vector of *observed* common factors, for each time $t = 1, 2, \ldots, T$, and let $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \ldots, \mathbf{d}_T]'$ be a $T \times M$ dimensional matrix. There are also *unobserved* common factors, denoted $\mathbf{f}_t$. Let $\mathbf{y} = [\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T]'$ be a $T \times N$ matrix of the dependent variables, across time and panel. Slicing this by panel, we have vectors of length T denoted $\mathbf{y}_i$ for $1 \leq i \leq N$. We can average over the panel, obtaining $\overline{\mathbf{Y}}$, a $T \times 1$ matrix (or vector). The covariates are given by $\mathbf{x}_t$ for $t = 1, 2, \ldots, T$, which is a $K \times N$ matrix. If we write the same over time, for each panel, we have a $T \times K$ matrix denoted $\mathbf{X}_i$ for the ith panel, $1 \leq i \leq N$. If we average across the panels, we obtain a K-vector (over time) denoted by $\overline{\mathbf{x}}_t$. Assembling these into a $T \times K$ matrix, we have

$$\overline{\mathbf{X}} = [\overline{\mathbf{x}}_1, \overline{\mathbf{x}}_2, \ldots, \overline{\mathbf{x}}_T]'.$$

Then a $T \times (M+1+K)$ dimensional matrix is defined via $\overline{\mathbf{H}} = [\mathbf{D}, \overline{\mathbf{Y}}, \overline{\mathbf{X}}]$. Regressing upon these quantities yields the projection matrix

$$\overline{\mathbf{M}} = I_T - \overline{\mathbf{H}} \left[\overline{\mathbf{H}}' \, \overline{\mathbf{H}} \right]^{-1} \overline{\mathbf{H}}'.$$

The CCE estimator of Pesaran (2006) is defined for each panel as follows: for $1 \leq i \leq N$,

$$\mathbf{b}_i = \left[\mathbf{X}_i' \, \overline{\mathbf{M}} \, \mathbf{X}_i \right]^{-1} \mathbf{X}_i' \, \overline{\mathbf{M}} \, \mathbf{y}_i. \tag{8}$$

The CCE is specialized to the CCE Mean Group (CCEMG) and CCE Pooled (CCEP) estimators, using $\mathbf{d}_t = 1$, as this is the only observed common factor in our model (1). Then the CCEMG estimator is defined as the straight average of the CCEs:

$$\mathbf{b}_{CCEMG} = N^{-1} \sum_{i=1}^{N} \mathbf{b}_i. \tag{9}$$

In contrast, the CCEP estimator averages over panels before doing the regression, using $\mathbf{d}_t = 1$:

$$\mathbf{b}_{CCEP} = \left[\sum_{i=1}^{N} \mathbf{X}_i' \,\overline{\mathbf{M}}\, \mathbf{X}_i\right]^{-1} \sum_{i=1}^{N} \mathbf{X}_i' \,\overline{\mathbf{M}}\, \mathbf{y}_i. \tag{10}$$

For comparison, we also construct the "infeasible" CCE estimator, which includes the unknown factor f_t in the regression of y_{it} on x_{it}; this provides an upper bound to the efficiency of the CCE estimators. The infeasible CCEMG is obtained by setting $\mathbf{d}_t' = [1, \mathbf{f}_t']$ instead of $\mathbf{d}_t = 1$; this estimator will be denoted by $\mathbf{b}_{INF}$.[1]

We also consider the OLS estimator that excludes the factor f_t. The OLS estimator shows the extent of bias and size distortions that can occur if the error cross-sectional dependence is present but ignored. Once we account for an overall mean, we obtain

$$\mathbf{b}_{OLS} = \left[\sum_{i=1}^{N} \mathbf{X}_i' \,\overline{\mathbf{P}}\, \mathbf{X}_i\right]^{-1} \sum_{i=1}^{N} \mathbf{X}_i' \,\overline{\mathbf{P}}\, \mathbf{y}_i, \tag{11}$$

where

$$\overline{\mathbf{P}} = I_T - \mathbf{D}\left[\mathbf{D}'\,\mathbf{D}\right]^{-1} \mathbf{D}'.$$

When $\mathbf{d}_t = 1$, this is called the pooled OLS estimator, and is denoted via $\mathbf{b}_{POOL}$; this is the estimator studied in the work of BGMS (2020).

Note that (11) resembles the CCEP estimator, where we have failed to first partial out the panel effects, only removing an overall mean. This construction leads to the Mean Group (MG) estimator, which is obtained by using $\overline{\mathbf{P}}$ in place of $\overline{\mathbf{M}}$ in (8), and then computing the CCEMG via (9); the estimator is denoted by $\mathbf{b}_{MG}$. $\mathbf{b}_{MG}$ is in spirit similar to BGMS (2020)'s approach by first running separate regressions for each individual and then aggregating. The only difference is that $\mathbf{b}_{MG}$ takes the mean of the coefficients obtained from the first step, while BGMS (2020) takes the median.

Finally, the aggregated model corresponds to applying OLS to $\overline{\mathbf{Y}}$ and $\overline{\mathbf{X}}$:

$$\mathbf{b}_{AGG} = \left[\overline{\mathbf{X}}'\,\overline{\mathbf{P}}\,\overline{\mathbf{X}}\right]^{-1} \overline{\mathbf{X}}'\,\overline{\mathbf{P}}\,\overline{\mathbf{y}}. \tag{12}$$

[1] Similarly we can construct the infeasible CCEP estimator.

This is the estimator proposed in CG (2015). Under the null hypothesis of no information frictions, $b_{AGG} = 0$, that is, the average forecast error is unpredictable using information dated t or earlier. In the presence of information rigidity, $b_{AGG} > 0$. This predictability in average forecast errors reflects the slow updating of information by some agents in Mankiw and Reis (2002)'s sticky-information model or the gradual adjustment of beliefs by all agents to new information in Sims (2003)'s noisy-information model.

Next, we explore the asymptotic properties of these estimators. As shown in Pesaran (2006), the CCEMG and CCEP estimators are unbiased (as is the INF method). In contrast, the AGG, MG and POOL estimators are biased, and it is possible to derive asymptotic bias expressions; see Appendix A for derivations. We provide formulas that are conditional on γ_i, τ_i, and θ_i. For short-hand, let $\sigma_f^2 = \text{Var}[f_t]$, $\sigma_i^2 = \text{Var}[\epsilon_{i,t}]$, and $\sigma_\cdot^2 = \text{Var}[\epsilon_{\cdot t}]$, where $\epsilon_{\cdot t} = \sum_{i=1}^N \epsilon_{it}$. For our derivations we assume that each θ_i is deterministic. The asymptotic biases are

$$\begin{aligned}
\text{Bias}_{AGG} &= \frac{\sum_{i=1}^N \gamma_i \sum_{i=1}^N \tau_i \sigma_f^2}{\left(\sum_{i=1}^N \tau_i\right)^2 \sigma_f^2 + \sigma_\cdot^2} \\
\text{Bias}_{MG} &= N^{-1} \sum_{i=1}^N \frac{\gamma_i \tau_i \sigma_f^2}{\tau_i^2 \sigma_f^2 + \sigma_i^2} \\
\text{Bias}_{POOL} &= \frac{\sum_{i=1}^N \gamma_i \tau_i \sigma_f^2}{\sum_{i=1}^N \tau_i^2 \sigma_f^2 + \sum_{i=1}^N \sigma_i^2}.
\end{aligned}$$

For all three estimators their biases depend on the variance of the common factor, σ_f^2. In the regression of cross-sectionally correlated forecast errors on forecast revisions, both MG and POOL estimators are biased. For the AGG estimator we see that the bias is negligible if either $\sum_{i=1}^N \gamma_i = 0$ or $\sum_{i=1}^N \tau_i = 0$; in either case, the bias will be low if σ_f^2 is small relative to $\sigma_\cdot^2$.

CG (2015) noted (in their Appendix A) that the AGG estimator is downward biased; this results from the presence of noise in the public information, leading to correlations between forecast revisions and the error term in the aggregate time series regression (of average forecast error on average forecast revision). Such results are explicable in terms of our framework: the particular state space structure in CG (2015) leads to a negative bias – see our Appendix B for details.

If there is no alternative to using the AGG method (say, when individual forecasts are not available), then it is desirable to reduce the bias. One possible approach is to utilize a high-pass filter on the covariate $\{x_{\cdot t}\}$ so as to suppress the common factor and accentuate $\{\epsilon_{\cdot t}\}$. (Such filtered covariates can be viewed as instrumental variables that the researcher constructs empirically.) This is certainly possible to do when the time series dynamics of $\{f_t\}$ and $\{\epsilon_{\cdot t}\}$ are known, and are somewhat distinct. This will have the effect of lowering the signal-to-noise ratio $\sigma_f^2/\sigma_{\cdot}^2$, thereby decreasing the magnitude of the bias in the AGG method.

3 Monte Carlo Study

We study the small-sample properties of the aforementioned estimators via Monte Carlo simulations. Our data generating process (DGP) is as follows. We simulate the data for $i = 1, \ldots, N$ and $t = 1, \ldots, T$ with stationary initializations. The individual fixed effect α_i is drawn from a normal distribution as $\alpha_i \sim$ i.i.d. $N(1, 1)$.

For the parameter of interest, β_i, we consider two cases: (i) homogeneous slope $\beta_i = \beta$ and (ii) heterogeneous slope $\beta_i = \beta + w_i$, $w_i \sim$ i.i.d. $N(0, 0.04)$. The null specification is $\beta = 0$, and under the alternative $\beta = .05$. In the homogeneous case all the panel slopes β_i are identical, so the group slope β is well-defined; in the heterogeneous case all the panel slopes β_i are different random variables, so the group slope is defined to be their common mean β.

For the factor loadings γ_i and τ_i, we consider strong cross-sectional dependence $\gamma_i \sim$ i.i.d. $N(1, 0.2)$ and $\tau_i \sim$ i.i.d. $N(1, 0.2)$, and weak cross-sectional dependence $\gamma_i \sim$ i.i.d. $N(0.5, 0.04)$ and $\tau_i \sim$ i.i.d. $N(0.5, 0.04)$. Also we allow the x variables to be independent ($\tau_i = 0$). If we wish to impose the constraint that $\sum_i \gamma_i = 0$, we take any given set of γ_i and subtract off their sample mean; this results in a negative cross-sectional correlation for the y variables.

The three parameters that govern time series correlations are drawn independently as $\rho = 0.5$, $\lambda_i \sim$ i.i.d. $U[0.05, 0.95]$ and $\theta_i \sim$ i.i.d. $U[0.05, 0.95]$. The error terms are specified as $v_t \sim$ i.i.d. $N(0, 1 - \rho^2)$, $u_{it} \sim$ i.i.d. $N(0, 1 - \lambda_i^2)$ and $\eta_{it} \sim$ i.i.d. $N(0, 1 - \theta_i^2)$.

We explore the properties of these estimators under several different scenarios: there are 12 DGPs based on the assumption of homogeneous slope.

- DGP 1: strong cross-sectional dependence in y only ($\gamma_i \sim$ i.i.d. $N(1, 0.2)$), but x are cross-sectionally independent ($\tau_i = 0$)
- DGP 2: strong cross-sectional dependence in y ($\gamma_i \sim$ i.i.d. $N(1, 0.2)$) and weak cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(0.5, 0.04)$)
- DGP 3: strong cross-sectional dependence in both y ($\gamma_i \sim$ i.i.d. $N(1, 0.2)$) and x ($\tau_i \sim$ i.i.d. $N(1, 0.2)$)
- DGP 4: weak cross-sectional dependence in y only ($\gamma_i \sim$ i.i.d. $N(0.5, 0.04)$), but x are cross-sectionally independent ($\tau_i = 0$)
- DGP 5: weak cross-sectional dependence in both y ($\gamma_i \sim$ i.i.d. $N(0.5, 0.04)$) and x ($\tau_i \sim$ i.i.d. $N(0.5, 0.04)$)
- DGP 6: weak cross-sectional dependence in y only ($\gamma_i \sim$ i.i.d. $N(0.5, 0.04)$), and strong cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(1, 0.2)$)
- DGP 7: zero bias constraint on y with high variance loadings ($\gamma_i = \delta_i - N^{-1} \sum_{j=1}^{N} \delta_j$, $\delta_i \sim$ i.i.d. $N(1, 0.2)$), but x are cross-sectionally independent ($\tau_i = 0$)
- DGP 8: zero bias constraint on y with high variance loadings ($\gamma_i = \delta_i - N^{-1} \sum_{j=1}^{N} \delta_j$, $\delta_i \sim$ i.i.d. $N(1, 0.2)$), and weak cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(0.5, 0.04)$)
- DGP 9: zero bias constraint on y with high variance loadings ($\gamma_i = \delta_i - N^{-1} \sum_{j=1}^{N} \delta_j$, $\delta_i \sim$ i.i.d. $N(1, 0.2)$), and strong cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(1, 0.2)$)

- DGP 10: zero bias constraint on y with low variance loadings ($\gamma_i = \delta_i - N^{-1}\sum_{j=1}^{N}\delta_j$, $\delta_i \sim$ i.i.d. $N(0.5, 0.04)$), but x are cross-sectionally independent ($\tau_i = 0$)

- DGP 11: zero bias constraint on y with low variance loadings ($\gamma_i = \delta_i - N^{-1}\sum_{j=1}^{N}\delta_j$, $\delta_i \sim$ i.i.d. $N(0.5, 0.04)$), and weak cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(0.5, 0.04)$)

- DGP 12: zero bias constraint on y with low variance loadings ($\gamma_i = \delta_i - N^{-1}\sum_{j=1}^{N}\delta_j$, $\delta_i \sim$ i.i.d. $N(0.5, 0.04)$), and strong cross-sectional dependence in x ($\tau_i \sim$ i.i.d. $N(1, 0.2)$)

Each experiment was replicated 50,000 times, first for $N = 50$, $T = 100$, and secondly for $N = 100$, $T = 50$. The simulation results are summarized in Tables 1 - 4, which provide estimates of bias, root mean squared error (RMSE), size in testing the hypothesis $\beta = 0$, and power assuming that $\beta = 0.05$.

Next, we compare the performance of six different estimators: CCEMG, INF, MG, CCEP, POOL, and AGG. CCEMG is the common correlated effects mean group estimator defined in equation (9), INF is the infeasible CCEMG estimator obtained by assuming that the common factor is known, and MG is the mean group estimator corresponding to first running separate regressions for each individual by ignoring the common factor, and then taking the mean of the coefficients obtained from the first step. CCEP is the common correlated effects pooled estimator defined in equation (10). POOL is the pooled OLS estimator without controlling for the panel structure and cross-sectional correlation. AGG is the OLS estimator from the aggregated time series regression as defined in equation (12).

Consider the bias and RMSE first. As shown in Tables 1 and 3, both MG and POOL estimators are substantially biased in the presence of cross-sectional correlation in the error term and individual-specific regressors (i.e. for DGPs 2-3 and 5-6). In contrast, the bias and RMSE of the CCE type estimators, including both CCEMG and CCEP, are very small and comparable to those of the infeasible estimator. For the homogeneous slope experiments, the CCEP is expected to be more efficient

Table 1: Bias and RMSE of estimators with homogeneous slope: $N = 50$, $T = 100$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Bias	-0.0002	-0.0001	-0.0002	-0.0001	0.0025	-0.0002
	RMSE	0.0205	0.0295	0.0196	0.0283	0.9601	0.0205
DGP 2	Bias	-0.0002	0.3853	-0.0001	0.3915	1.8586	-0.0001
	RMSE	0.0204	0.3908	0.0196	0.3971	1.8661	0.0204
DGP 3	Bias	0.0001	0.4514	0.0001	0.4557	0.9849	0.0001
	RMSE	0.0203	0.4548	0.0195	0.4587	0.9891	0.0203
DGP 4	Bias	0.0000	-0.0000	-0.0000	-0.0001	-0.0029	0.0000
	RMSE	0.0204	0.0226	0.0196	0.0218	0.4930	0.0204
DGP 5	Bias	0.0002	0.1931	0.0001	0.1962	0.9298	0.0002
	RMSE	0.0205	0.1964	0.0197	0.1995	0.9337	0.0206
DGP 6	Bias	-0.0002	0.2256	-0.0002	0.2278	0.4923	-0.0002
	RMSE	0.0204	0.2275	0.0196	0.2294	0.4945	0.0205
DGP 7	Bias	-0.0001	-0.0001	-0.0001	-0.0001	-0.0011	-0.0000
	RMSE	0.0222	0.0219	0.0214	0.0212	0.1372	0.0204
DGP 8	Bias	0.0001	0.0001	0.0001	0.0000	0.0001	0.0002
	RMSE	0.0205	0.0199	0.0197	0.0206	0.0357	0.0204
DGP 9	Bias	0.0001	0.0000	0.0000	-0.0000	0.0001	0.0000
	RMSE	0.0204	0.0162	0.0196	0.0185	0.0184	0.0204
DGP 10	Bias	0.0002	0.0002	0.0002	0.0002	0.0005	0.0002
	RMSE	0.0208	0.0205	0.0200	0.0198	0.1369	0.0205
DGP 11	Bias	-0.0001	-0.0001	-0.0001	-0.0001	0.0001	-0.0001
	RMSE	0.0204	0.0180	0.0196	0.0177	0.0357	0.0204
DGP 12	Bias	0.0001	-0.0000	0.0001	-0.0000	-0.0000	0.0001
	RMSE	0.0203	0.0145	0.0196	0.0140	0.0183	0.0204

Note: This table shows the bias and root mean squared error (RMSE) based on $50,000$ replications for the homogeneous case. CCEMG is the common correlated effects mean group estimator defined in equation (9), INF is the infeasible CCEMG estimator by assuming that the common factor is known, and MG is the mean group estimator by first running separate regressions for each individual by ignoring the common factor, and then taking the mean of the coefficients obtained from the first step. CCEP is the common correlated effects pooled estimator defined in equation (10). POOL is the pooled OLS estimator without controlling for the panel structure and cross-sectional correlation. AGG is the OLS estimator from the aggregated time series regression as defined in equation (12).

Table 2: Size and power of estimators with homogeneous slope: $N = 50$, $T = 100$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Size	0.0569	0.0552	0.0571	0.0549	0.1386	0.0562
	Power	0.6972	0.4162	0.7272	0.4375	0.1381	0.6979
DGP 2	Size	0.0571	1.0000	0.0580	1.0000	1.0000	0.0576
	Power	0.6999	1.0000	0.7301	1.0000	1.0000	0.6987
DGP 3	Size	0.0574	1.0000	0.0563	1.0000	1.0000	0.0561
	Power	0.6998	1.0000	0.7315	1.0000	1.0000	0.6979
DGP 4	Size	0.0571	0.0543	0.0572	0.0539	0.1357	0.0574
	Power	0.6956	0.6027	0.7254	0.6309	0.1369	0.6954
DGP 5	Size	0.0581	1.0000	0.0586	1.0000	1.0000	0.0588
	Power	0.6986	1.0000	0.7297	1.0000	1.0000	0.6969
DGP 6	Size	0.0568	1.0000	0.0583	1.0000	1.0000	0.0573
	Power	0.6986	1.0000	0.7283	1.0000	1.0000	0.6958
DGP 7	Size	0.0559	0.0544	0.0575	0.0549	0.1508	0.0566
	Power	0.6275	0.6344	0.6598	0.6633	0.1773	0.6975
DGP 8	Size	0.0574	0.0039	0.0570	0.0041	0.1364	0.0575
	Power	0.6938	0.3014	0.7239	0.2899	0.4680	0.6968
DGP 9	Size	0.0560	0.0003	0.0573	0.0006	0.1351	0.0552
	Power	0.7027	0.2199	0.7328	0.2119	0.8847	0.7001
DGP 10	Size	0.0589	0.0561	0.0583	0.0558	0.1496	0.0577
	Power	0.6846	0.6886	0.7157	0.7170	0.1762	0.6967
DGP 11	Size	0.0564	0.0248	0.0571	0.0258	0.1370	0.0569
	Power	0.6994	0.6860	0.7305	0.6948	0.4676	0.6961
DGP 12	Size	0.0571	0.0115	0.0572	0.0105	0.1343	0.0575
	Power	0.6995	0.7942	0.7287	0.8085	0.8855	0.6949

Note: This table reports the proportion of rejections of the null hypothesis that $\beta = 0$, when $\beta = 0$ (for size of various estimators) or $\beta = 0.05$ (for power of various estimators), based on $50,000$ replications for the homogeneous case. For the description of different estimators, see the note to Table 1.

than the CCEMG, and this is corroborated by the results in these tables. Indeed, the CCEP estimator even dominates the infeasible estimator. As expected, the AGG estimator is unbiased when the error terms are cross-sectionally independent (i.e., $\sum_{i=1}^{N} \gamma_i = 0$ for DGPs 7-12) or when the individual-specific regressors x_{it} are cross-sectionally independent for DGPs 1 and 4. In more general cases (e.g. when forecasters have access to the same public information with noise), however, the AGG estimator exhibits large biases.

Using the asymptotic bias formulas of Section 2, it is possible to obtain explicit values in the case of the MG and POOL estimators. This is

Table 3: Bias and RMSE of estimators with homogeneous slope: $N = 100$, $T = 50$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Bias	0.0001	-0.0000	0.0001	0.0000	-0.0031	0.0001
	RMSE	0.0206	0.0300	0.0191	0.0280	1.9104	0.0206
DGP 2	Bias	-0.0000	0.3969	0.0000	0.3959	1.9283	-0.0000
	RMSE	0.0206	0.4051	0.0191	0.4045	1.9331	0.0207
DGP 3	Bias	0.0000	0.4580	0.0000	0.4558	0.9924	0.0000
	RMSE	0.0205	0.4625	0.0191	0.4596	0.9947	0.0206
DGP 4	Bias	-0.0000	-0.0001	-0.0000	-0.0001	-0.0006	-0.0000
	RMSE	0.0206	0.0230	0.0191	0.0214	0.9693	0.0207
DGP 5	Bias	0.0001	0.1987	0.0000	0.1981	0.9642	0.0001
	RMSE	0.0205	0.2033	0.0190	0.2029	0.9670	0.0206
DGP 6	Bias	0.0002	0.2293	0.0001	0.2281	0.4963	0.0001
	RMSE	0.0206	0.2318	0.0191	0.2302	0.4976	0.0207
DGP 7	Bias	0.0001	0.0000	0.0000	-0.0000	-0.0001	0.0001
	RMSE	0.0226	0.0223	0.0209	0.0208	0.1900	0.0207
DGP 8	Bias	-0.0000	-0.0000	0.0000	0.0001	0.0002	0.0000
	RMSE	0.0206	0.0194	0.0192	0.0191	0.0358	0.0207
DGP 9	Bias	-0.0002	-0.0001	-0.0002	-0.0001	-0.0000	-0.0001
	RMSE	0.0205	0.0155	0.0191	0.0159	0.0182	0.0206
DGP 10	Bias	0.0001	0.0001	0.0001	0.0001	-0.0003	0.0001
	RMSE	0.0209	0.0206	0.0193	0.0192	0.1894	0.0206
DGP 11	Bias	0.0002	0.0002	0.0001	0.0001	0.0002	0.0002
	RMSE	0.0206	0.0178	0.0191	0.0170	0.0360	0.0207
DGP 12	Bias	0.0000	0.0000	0.0000	0.0000	-0.0000	0.0000
	RMSE	0.0206	0.0143	0.0191	0.0132	0.0181	0.0207

Note: This table shows the bias and root mean squared error (RMSE) based on 50,000 replications for the homogeneous case. For the description of different estimators, see the note to Table 1.

because $\sigma_f^2 = 1$ and $\sigma_i^2 = 1$ for each i. All biases are zero for DGPs 7 through 12, and are also zero for DGPs 1 and 4. Both POOL and MG have asymptotic bias of 2/5 for DGP 2, and 1/2 for DGP 3; for DGPs 5 and 6 both estimators have biases of 1/5 and 1/4 respectively. The quantity $\sigma_\cdot^2$ cannot be exactly determined in our simulation setup because we randomly generated each θ_i. But if we assume that $\sigma_\cdot^2/N^2$ is much smaller than one, then the asymptotic bias of the AGG estimator is approximately 2 for DGP 2, 1 for DGPs 3 and 5, and 1/2 for DGP 6. These asymptotic bias results explain the observed biases in our simulations.

Turning to the size and power reported in Tables 2 and 4, the test results for both CCEMG and CCEP estimators have the correct size for

Table 4: Size and power of estimators with homogeneous slope: $N = 100$, $T = 50$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Size	0.0535	0.0527	0.0535	0.0520	0.1328	0.0528
	Power	0.6866	0.4030	0.7489	0.4466	0.1352	0.6828
DGP 2	Size	0.0521	1.0000	0.0543	1.0000	1.0000	0.0527
	Power	0.6838	1.0000	0.7465	1.0000	1.0000	0.6803
DGP 3	Size	0.0524	1.0000	0.0527	1.0000	1.0000	0.0533
	Power	0.6877	1.0000	0.7477	1.0000	1.0000	0.6818
DGP 4	Size	0.0536	0.0519	0.0529	0.0517	0.1329	0.0533
	Power	0.6842	0.5909	0.7470	0.6451	0.1328	0.6813
DGP 5	Size	0.0526	1.0000	0.0516	1.0000	1.0000	0.0518
	Power	0.6889	1.0000	0.7482	1.0000	1.0000	0.6836
DGP 6	Size	0.0531	1.0000	0.0538	1.0000	1.0000	0.0531
	Power	0.6882	1.0000	0.7466	1.0000	1.0000	0.6829
DGP 7	Size	0.0540	0.0519	0.0543	0.0521	0.1414	0.0525
	Power	0.6114	0.6204	0.6747	0.6768	0.1545	0.6808
DGP 8	Size	0.0525	0.0100	0.0531	0.0097	0.1335	0.0531
	Power	0.6845	0.4746	0.7454	0.4860	0.4639	0.6831
DGP 9	Size	0.0521	0.0016	0.0541	0.0025	0.1320	0.0520
	Power	0.6860	0.4976	0.7448	0.4901	0.8838	0.6801
DGP 10	Size	0.0514	0.0517	0.0536	0.0528	0.1406	0.0520
	Power	0.6738	0.6816	0.7362	0.7357	0.1539	0.6830
DGP 11	Size	0.0539	0.0328	0.0544	0.0333	0.1333	0.0544
	Power	0.6874	0.7435	0.7471	0.7779	0.4659	0.6830
DGP 12	Size	0.0533	0.0217	0.0528	0.0186	0.1335	0.0532
	Power	0.6863	0.8764	0.7477	0.9132	0.8824	0.6814

Note: This table reports the proportion of rejections of the null hypothesis that $\beta = 0$, when $\beta = 0$ (for size of various estimators) or $\beta = 0.05$ (for power of various estimators), based on $50,000$ replications for the homogeneous case. For the description of different estimators, see the note to Table 1.

$N, T \geq 50$. Perhaps not surprisingly, given the homogeneous nature of the alternative, the CCEP estimator is more powerful than the CCEMG estimator, particularly for relatively large N. In contrast, both MG and POOL estimators are severely oversized in the presence of cross-sectional correlation (i.e. for DGPs 2-3 and 5-6), but are correctly sized when the individual-specific regressors are cross-sectionally independent. Finally, the AGG estimator displays size distortions in all cases. Upon further investigation, we find that the variance estimator for the AGG method is too small, leading to an inflated value of the studentized statistic (and hence too many type I errors). These problems seem to arise because the regression errors have positive serial dependence, and the OLS-based variance estimators do not take this into account.

The simulations based on heterogeneous slopes give almost the same results. To save space, they are reported in Appendix C. Overall, both CCEMG and CCEP estimators perform very well: they have the correct size, satisfactory power for moderately large N and T, and very small root mean squared errors, which are comparable to those of the infeasible estimators. By contrast, the other three estimators, including MG, POOL, and AGG, display large biases and suffer from severe size distortions in the presence of cross-sectional correlation.

4 Rationality in Macroeconomic Expectations

We use the individual and consensus forecasts from the Survey of Professional Forecasters (SPF) currently run by the Federal Reserve Bank of Philadelphia covering 1968.Q4 through 2016.Q4[2]. To facilitate the comparison to BGMS (2020), we use an annual forecast horizon. For GDP and inflation, we transform the level of these variables into implied growth rates from quarter $t-1$ to quarter $t+3$. For variables such as unemployment rate and interest rates, we study the level in quarter $t+3$. We compute consensus forecasts as the mean of individual forecasts.

The original data set has 333 agents and 193 quarters. About 90% of the data is missing, so we use the following techniques to refine the data. First, we split the data into two distinct time spans: (1) 1968.Q4 through 1990.Q4 and (2) 1992.Q1 through 2016.Q4. The American Statistical Association and the National Bureau of Economic Research initiated the survey in 1968Q4. Due to a rapidly declining participation rate in the late 1980s, the Federal Reserve Bank of Philadelphia took over the survey in 1990 with a new infusion of forecasters. Our decision for sample split is driven by this structural change in the survey. Also for this first

[2]Federal Reserve Bank of Philadelphia, Survey of Professional Forecasters: 1968.Q4 - 2016.Q4 (accessed February 9, 2021), https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/individual-forecasts

span, we restrict to the six variables: nominal GDP, real GDP, GDP price index, industrial production, housing starts, and unemployment. For the second span we retain all 15 pairs of variables. In addition to the six variables above, we also consider forecasts for consumer price index, real consumption, real nonresidential investment, real residential investment, real federal government consumption, real state and local government consumption, three-month Treasury rate, ten-year Treasury rate, and AAA corporate bond rate.

Further, within each of these two data subsets we consider those agents who have responded at least 20% of the time, i.e. the proportion of missing values for such agents is at most 80% across all times and variables. This gives $N = 38$ agents and $T = 89$ quarters for the first subset and $N = 36$ agents and $T = 100$ quarters for the second subset, which are roughly in the range of sample sizes for our first simulation.

As a first step, we replace missing values using the following technique. For each variable, let y denote a vector corresponding to the $T \times N$ data matrix, with all times for the first agent followed by all times for the second agent, etc. We also construct dummy regressors consisting of the means across the agents (for each time), and the means across time (for each agent). For this second covariate, we subtract the grand mean (average over agents and time) for the time means. Let this regressor matrix be denoted X. Note that in constructing X, which is $NT \times 2$-dimensional, we must handle missing values to construct the panel means and time means: we simply omit any missing values to compute these averages, and if all the inputs happen to be missing, then that particular average is replaced by the grand mean.

Next, consider a linear model $y = X\beta + \epsilon$, where β is a bivariate vector and ϵ are the errors. Such a model is considered in Lahiri, et al. (2021). Let J and K be selection matrices such that Jy is observed and Ky consists of all the missing values in y. Then

$$Jy = JX\beta + J\epsilon$$

is an implied regression equation where, all the dependent variables are observed. So we can estimate β on the basis of this fully observed regression, via

$$\widehat{\beta} = [(JX)'(JX)]^{-1}(JX)'Jy.$$

This estimate can be computed because there are no missing values involved in JX or Jy. To obtain imputations for Ky we use the predictors

$$\widehat{Ky} = K\widehat{y} = KX\widehat{\beta} = KX[(JX)'(JX)]^{-1}(JX)'Jy.$$

We apply this technique to impute missing values to the SPF dataset. Individual forecast errors (y_{it}) are calculated as actual values minus individual forecasts. We use the first-released actual values in real time from the Real Time Data Set for Macroeconomists provided by the Federal Reserve Bank of Philadelphia[3]. Individual forecast revisions (x_{it}) are computed as individual i's forecasts made in quarter t minus her forecasts made in quarter $t-1$. We obtain consensus forecast errors and forecast revisions as the average of the corresponding individual forecast errors and revisions.

Columns 1-5 in Table 5 report the coefficient estimates from the regression of forecast errors on forecast revisions and the associated p-values for the t-statistics for the CCEMG, MG, CCEP, POOL, and AGG estimators. The last two columns report the p-values from testing the null hypothesis that slopes are homogeneous (SL) and that there is no cross-sectional dependence in the error term (CD).

The SL test is from Pesaran and Yamagata (2008)'s adjusted version of the slope homogeneity test, which is robust to non-normal errors in large panels. For about half of the cases, the SL test rejects the null of slope homogeneity and supports mean group estimators rather than pooled estimators. Pesaran (2021)'s CD test is based on the average of pair-wise correlation coefficients of the OLS residuals from the individual regressions in the panel. The CD test results unanimously reject the independence null hypothesis and point to the widespread cross-sectional correlation in the error terms of information rigidity regressions.

[3]Federal Reserve Bank of Philadelphia, Real-Time Data Set for Macroeconomists: 1968.Q4 - 2016.Q4 (accessed February 9, 2021), `https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/real-time-data-set-for-macroeconomists`

Table 5: Test for information rigidity

Variable	CCEMG	MG	CCEP	POOL	AGG	SL	CD
Panel A: 1968.Q4-1990.Q4 ($N = 38$, $T = 89$)							
NGDP	-0.41 (0.00)	0.21 (0.00)	-0.42 (0.00)	0.14 (0.01)	0.62 (0.02)	0.71	0.00
RGDP	-0.39 (0.00)	0.11 (0.01)	-0.37 (0.00)	0.02 (0.77)	0.42 (0.14)	0.18	0.00
PGDP	-0.44 (0.00)	0.81 (0.00)	-0.38 (0.00)	0.66 (0.00)	1.53 (0.00)	0.00	0.00
INDP	-0.41 (0.00)	0.16 (0.00)	-0.41 (0.00)	0.14 (0.00)	0.65 (0.06)	0.05	0.00
HOUS	-0.38 (0.00)	0.19 (0.00)	-0.41 (0.00)	0.17 (0.00)	0.51 (0.11)	0.04	0.00
UNEM	-0.34 (0.00)	0.30 (0.00)	-0.34 (0.00)	0.30 (0.00)	0.52 (0.06)	0.00	0.00
Panel B: 1992.Q1-2016.Q4 ($N = 36$, $T = 100$)							
NGDP	-0.46 (0.00)	0.21 (0.00)	-0.50 (0.00)	0.13 (0.04)	0.61 (0.02)	0.00	0.00
RGDP	-0.44 (0.00)	0.12 (0.00)	-0.46 (0.00)	0.08 (0.08)	0.32 (0.23)	0.71	0.00
PGDP	-0.51 (0.00)	-0.21 (0.00)	-0.53 (0.00)	-0.25 (0.00)	0.23 (0.41)	0.18	0.00
CPI	-0.55 (0.00)	-0.11 (0.00)	-0.53 (0.00)	-0.16 (0.00)	0.22 (0.48)	0.98	0.00
RCON	-0.48 (0.00)	0.06 (0.12)	-0.48 (0.00)	-0.02 (0.74)	0.38 (0.19)	0.61	0.00
INDP	-0.52 (0.00)	0.29 (0.00)	-0.50 (0.00)	0.21 (0.00)	0.75 (0.05)	0.71	0.00
RNRE	-0.50 (0.00)	0.53 (0.00)	-0.52 (0.00)	0.43 (0.00)	1.19 (0.00)	0.00	0.00
RRES	-0.37 (0.00)	0.64 (0.00)	-0.39 (0.00)	0.46 (0.00)	1.75 (0.00)	0.00	0.00
RFGC	-0.56 (0.00)	-0.50 (0.00)	-0.56 (0.00)	-0.50 (0.00)	-0.40 (0.22)	0.13	0.00
RGSL	-0.50 (0.00)	-0.09 (0.19)	-0.53 (0.00)	-0.25 (0.00)	0.80 (0.03)	0.00	0.00
HOUS	-0.45 (0.00)	-0.10 (0.05)	-0.48 (0.00)	-0.26 (0.00)	0.25 (0.46)	0.01	0.00
UNEM	-0.29 (0.00)	0.72 (0.00)	-0.30 (0.00)	0.70 (0.00)	1.03 (0.00)	0.02	0.00
tb3m	-0.26 (0.00)	0.56 (0.00)	-0.24 (0.00)	0.53 (0.00)	0.79 (0.00)	0.38	0.00
tn10y	-0.35 (0.00)	-0.11 (0.00)	-0.34 (0.00)	-0.12 (0.00)	-0.01 (0.95)	0.00	0.00
AAA	-0.37 (0.00)	-0.18 (0.00)	-0.38 (0.00)	-0.20 (0.00)	-0.03 (0.87)	0.08	0.00

Note: Columns 1-5 of this table show the coefficient estimates (with p-values in parentheses) from the regression of forecast errors on forecast revisions. See the note to Table 1 for the description of the CCEMG, MG, CCEP, POOL, and AGG estimators. The last two columns report the p-values from testing the null hypothesis that slopes are homogeneous (SL) and the null that there is no cross-sectional dependence in the error term (CD). Data source: Philadelphia Fed Survey of Professional Forecasters (SPF) forecasts for nominal GDP (NGDP), real GDP (RGDP), GDP price index (PGDP), industrial production (INDP), housing start (HOUS), unemployment (UNEM), consumer price index (CPI), real consumption (RCON), real nonresidential investment (RNRE), real residential investment (RRES), real federal government consumption (RFGC), real state and local government consumption (RGSL), three-month Treasury rate (tb3m), ten-year Treasury rate (tn10y) and AAA corporate bond rate (AAA).

Considering consensus forecasts first, the slope coefficients from the AGG estimator are positive for all 6 variables in the first subsample and statistically significant at the 10 percent level for 4 of them. In the second subsample, the AGG estimator is positive for 12 variables, with 7 of them being statistically significant, and negative, albeit insignificant, for 3 variables. These potentially biased results suggest that consensus forecasts of macroeconomic variables underreact to new information, consistent with CG (2015)'s and BGMS (2020)'s findings.

For individual forecasts, the results from the MG and POOL estimators are mixed. For the first subsample, both estimators give positive coefficients, suggesting underreaction to news. For the second subsample, however, these two estimators yield negative and statistically significant slope coefficients for 6 and 7 variables, respectively. An important caveat is in order: these estimates are most likely biased since both MG and POOL estimators ignore the presence of cross-sectional correlation, as indicated by the CD test.

In contrast, the CCEMG estimator that allows for slope heterogeneity and cross-sectional correlation gives consistently negative slope coefficients, statistically significant at the 1 percent level for all variables in both subsamples. The CCEP estimator, by assuming slope homogeneity, yields nearly identical results. The augmented information rigidity test suggests that individual professional forecasters overreact to news in their macroeconomic expectations.

One potential concern is that our empirical results might be dominated by the missing values in the SPF dataset. To address this concern, we have performed additional simulations where a proportion p of both dependent and independent variables are missing at random, with p ranging from 10% to 80%. Table C.5 reports the results for the bias, RMSE, and size for the homogeneous slope case. Missing values have a negligible effect on our CCEMG and CCEP estimators; thus, our main empirical finding of overreaction at the individual forecaster level still holds. As the percentage of missing data increases, the bias and RMSE for MG and POOL estimators get slightly larger, while the AGG estimator becomes marginally less biased.

5 Conclusion

This article studies the problem of regressing forecast errors on forecast revisions at both individual and aggregate levels. The approach of “average then estimate” using consensus forecasts will be unbiased when the underlying error terms are cross-sectionally independent or when forecast revisions are cross-sectionally independent. In more general cases where forecasters have access to noisy public information, however, this approach exhibits large biases. By contrast, the approach of “estimate then average” using individual forecasts will be unbiased in the absence of cross-sectional correlation in forecast errors or forecast revisions. Apart from these conditions, bias and size distortions can arise.

In order to remove potential biases one can utilize common correlated effects estimators. In particular, the common correlated effects pooled estimator is effective when the slope is homogeneous. If the slope is heterogeneous, the common correlated effects mean group estimator is recommended; this can also be used in the case of a homogeneous slope, but is slightly less efficient than the pooled estimator. Because such decisions about which estimator to use are contingent on knowing the slope heterogeneity and the cross-sectional dependence in the error term, we recommend checking these two conditions in practice.

In testing for rationality in the U.S. Survey of Professional Forecasters dataset, the approach of “average then estimate” provides positive, potentially biased, estimates of slope coefficients, implying that consensus forecasts underreact to new information relative to full-information rational expectations. The approach of “estimate then average” using individual forecasts gives a mixture of positive and negative estimates. In contrast, the common correlated effects estimators, including both mean group and pooled, always yield negative slope estimates and indicate that professional forecasters on average overreact to news. This discrepancy in results can be explained through the aforementioned biases to which the simple aggregation or pooled estimators are subject. Finally, our results that individual forecasts overreact to news but consensus forecasts might underreact suggest that insights based on individual behavior do not necessarily carry over to consensus forecasts. The “estimate then average” and "pool" estimators of BGMS (2020) would be a valid strategy in testing an information frictions hypothesis only by accounting for cross-sectional and time series correlations in a panel data setting. The "average then estimate" approach needs to be generalized to allow for common noise in public information, and we leave it for future research.

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Appendix A Asymptotic Bias Derivations

Here we derive asymptotic bias expressions for the AGG, MG, and POOL estimators conditional on the γ_i, τ_i, and θ_i being known. The derivations only use the Law of Large Numbers, and we let $\xrightarrow{P}$ denote convergence in probability. We also use $\approx$ to denote expressions that differ by a sequence tending to zero in probability.

Beginning with AGG, note that by regressing $y_{\cdot t}$ on $x_{\cdot t}$ and a constant, we can remove constant effects asymptotically, obtaining

$$\begin{aligned}\mathbf{b}_{AGG} &\approx \frac{\sum_t \left(\beta\, x_{\cdot t} + \sum_{i=1}^N \gamma_i\, f_t + \xi_{\cdot t}\right) x_{\cdot t}}{\sum_t x_{\cdot t}^2} \\ &= \beta + \frac{\sum_{i=1}^N \gamma_i\, \sum_t f_t x_{\cdot t} + \sum_t \xi_{\cdot t} x_{\cdot t}}{\sum_t x_{\cdot t}^2}.\end{aligned}$$

Next, $x_{\cdot t}$ and $\xi_{\cdot t}$ are uncorrelated with each other, and $\epsilon_{\cdot t}$ is uncorrelated with f_t; therefore as sample size T tends to infinity

$$\mathbf{b}_{AGG} \xrightarrow{P} \beta + \frac{\sum_{i=1}^N \gamma_i\, \sum_{i=1}^N \tau_i\, \sigma_f^2}{\left(\sum_{i=1}^N \tau_i\right)^2 \sigma_f^2 + \sigma_\cdot^2},$$

since $\sum_t f_t^2 \xrightarrow{P} \sigma_f^2$ and $\sum_t \epsilon_{\cdot t} \xrightarrow{P} \sigma_\cdot^2$.

For the MG estimator, the derivation for the ith slope parameter is similar to the aggregated case. Once the panel mean has been accounted for,

$$\begin{aligned}\mathbf{b}_i &\approx \frac{\sum_t (\beta\, x_{it} + \gamma_i\, f_t + \xi_{it})\, x_{it}}{\sum_t x_{it}^2} \\ &= \beta + \frac{\gamma_i\, \sum_t f_t x_{it} + \sum_t \xi_{it} x_{it}}{\sum_t x_{it}^2}\end{aligned}$$

for each $1 \le i \le N$. For large sample size T we have

$$\mathbf{b}_i \xrightarrow{P} \beta + \frac{\gamma_i \tau_i\, \sigma_f^2}{\tau_i^2\, \sigma_f^2 + \sigma_i^2},$$

and now the asymptotic bias expression follows from averaging over the panels, using (9).

For the POOL estimator we ignore the panel structure and remove the overall mean; for the dependent variable this is $(NT)^{-1}\sum_{i,t} y_{it} \approx N^{-1}\sum_{i=1}^{N} \alpha_i$ for large T. Since the independent variables have mean zero, asymptotically we can ignore the effect of de-meaning them, and hence

$$\begin{aligned}\mathbf{b}_{POOL} &\approx \frac{\sum_{i,t}\left(\alpha_i - N^{-1}\sum_{j=1}^{N}\alpha_j + \beta\, x_{it} + \gamma_i\, f_t + \xi_{it}\right) x_{it}}{\sum_{i,t} x_{it}^2} \\ &= \beta + \frac{\sum_i\left(\alpha_i - N^{-1}\sum_{j=1}^{N}\alpha_j\right)\sum_t x_{it} + \sum_{i,t}\gamma_i\, f_t x_{it} + \sum_{i,t}\xi_{it}x_{it}}{\sum_t x_{\cdot t}^2}.\end{aligned}$$

Now $T^{-1}\sum_t x_{it} \xrightarrow{P} 0$ for each i, and we see that

$$\mathbf{b}_{POOL} \xrightarrow{P} \beta + \frac{\sum_{i=1}^{N}\gamma_i\tau_i\,\sigma_f^2}{\sum_{i=1}^{N}\tau_i^2\,\sigma_f^2 + \sum_{i=1}^{N}\sigma_i^2}.$$

Appendix B AGG Estimator under Coibion and Gorodnichenko (2015, CG hereafter)'s Framework

We now make an application of the AGG results to the framework of CG (2015), preserving the notation used in their paper (which conflicts somewhat with symbols we have used above). In their state space framework, a public signal $\{\pi_t\}$ is observed by agent i with both public and private noise $\{e_t\}$ and $\{\omega_{it}\}$. The signal is an autoregressive process:

$$\pi_t = \rho\, \pi_{t-1} + \nu_t,$$

with $-1 < \rho < 1$ and $\{\nu_t\}$ an i.i.d. Gaussian sequence with mean zero and variance Σ_ν. The observed process for agent i is

$$y_{i,t} = \pi_t + e_t + \omega_{it},$$

where $\{e_t\}$ is i.i.d. Gaussian with mean zero and variance Σ_e, and for each i, $\{\omega_{it}\}$ is i.i.d. Gaussian with mean zero and common variance Σ_ω. All the latent processes are independent of each other.

We use a common notation for conditional expectations given past values of the observed process $\{y_{i,t}\}$, viz. $\pi_{t|t}(i)$ for the projection of the signal onto present and past values of $\{y_{i,t}\}$. Similarly, a forecast of the signal is denoted $\pi_{t|t-1}(i)$. A further averaging over the forecasters is denoted by $\overline{\pi}_{t|t}$ and $\overline{\pi}_{t|t-1}$, and this operation can be viewed as computing an expectation conditional on $\{\pi_t\}$ and $\{e_t\}$, thereby eradicating the presence of any $\omega_{i,t}$. To begin,

$$\pi_{t|t}(i) = \pi_{t|t-1}(i) + G\left(y_t - y_{t|t-1}(i)\right)$$

expresses the classic formula for conditional expectations, where the information set on the left hand side includes $A_{t-1} = \{\ldots, y_{i,t-2}, y_{i,t-1}\}$ as well as $y_{i,t}$, and the right hand side separates the two contributions. Hence G is just the covariance between π_t and the new information $y_{i,t} - y_{t|t-1}(i)$, normalized by the variance of the latter – this quantity G is called the

Kalman gain.

Next, because e_t and $\omega_{i,t}$ are independent of the old information A_{t-1}, we find $y_{t|t-1}(i) = \pi_{t|t-1}(i)$, and it follows that

$$\pi_{t|t}(i) = (1 - G)\, \pi_{t|t-1}(i) + G\, y_t.$$

If we average across the forecasters, we obtain

$$\overline{\pi}_{t|t} = (1 - G)\, \overline{\pi}_{t|t-1} + G\, (\pi_t + e_t).$$

We need a few other expressions. Considering h-step ahead forecasting ($h \geq 1$), we can recursively expand the autoregressive signal to obtain

$$\pi_{t+h} = \rho^h\, \pi_t + \rho^{h-1}\, \nu_{t+1} + \ldots + \rho\, \nu_{t+h-1} + \nu_{t+h}.$$

Gathering all but the first term on the right hand side, CG (2015) defines $\nu_{t+h,t} = \sum_{k=0}^{h-1} \rho^k \nu_{t+h-k}$. If we now project onto present and past data, we obtain

$$\pi_{t+h|t}(i) = \rho^h\, \pi_{t|t}(i),$$

from which $\overline{\pi}_{t+h|t} = \rho^h\, \overline{\pi}_{t|t}$. Hence,

$$\pi_{t+h} - \overline{\pi}_{t+h|t} = \nu_{t+h,t} + \rho^h\, \left(\pi_t - \overline{\pi}_{t|t}\right).$$

Moreover, using previous calculations we obtain

$$\begin{aligned}\overline{\pi}_{t+h|t} - (1 - G)\, \overline{\pi}_{t+h|t-1} &= \rho^h\, \overline{\pi}_{t|t} - (1 - G)\, \rho^h\, \overline{\pi}_{t|t-1} \\ &= \rho^h\, G\, (\pi_t + e_t).\end{aligned}$$

Using these expressions,

$$\begin{aligned}G\, \left(\pi_{t+h} - \overline{\pi}_{t+h|t}\right) &= G\, \nu_{t+h,t} + G\, \rho^h\, \left(\pi_t - \overline{\pi}_{t|t}\right) \\ &= G\, \nu_{t+h,t} - \rho^h\, G\, e_t + \overline{\pi}_{t+h|t} - (1 - G)\, \overline{\pi}_{t+h|t-1} - G\, \overline{\pi}_{t+h|t} \\ &= G\, (\nu_{t+h,t} - \rho^h\, e_t) + (1 - G)\, \left(\overline{\pi}_{t+h|t} - \overline{\pi}_{t+h|t-1}\right).\end{aligned}$$

Dividing through by G now yields

$$\left(\pi_{t+h} - \overline{\pi}_{t+h|t}\right) = \frac{1-G}{G} \left(\overline{\pi}_{t+h|t} - \overline{\pi}_{t+h|t-1}\right) + (\nu_{t+h,t} - \rho^h \, e_t),$$

which resembles a regression of h-step ahead average forecast errors on h-step ahead average forecast revisions. (Note that in Appendix A of CG (2015), the factor ρ^h that multiplies e_t has been omitted.) We can now match this regression equation to the aggregation framework (7) of our paper. For the dependent variable (i.e. average forecast error), our dynamic factor f_t corresponds to the public noise e_t, and $\sum_{i=1}^N \gamma_i = -\rho^h$, whereas $\xi_{\cdot t}$ corresponds to $\nu_{t+h,t}$. Also, $\sum_{i=1}^N \alpha_i = 0$ and $\beta = (1-G)/G$. Turning to the covariate (i.e. average forecast revision), we see that the same dynamic factor f_t is present, and $\sum_{i=1}^N \tau_i = \rho^h G$, whereas $\epsilon_{\cdot t}$ corresponds to the remaining independent portion

$$\rho^h \, (1-G) \, \overline{\pi}_{t|t-1} + \rho^h \, G \, \pi_t - \overline{\pi}_{t+h|t-1}.$$

Now we can plug into the formulas for the asymptotic bias of the AGG estimator, and find that the numerator equals $-\rho^{2h} \, G \, \Sigma_e$. (In CG (2015), the bias expression involves ρ^h instead of ρ^{2h}, because of their typo for the coefficient of e_t mentioned above.) The denominator is the variance of the covariate, and is always positive; since ρ^{2h} is non-negative for any $-1 < \rho < 1$ and any $h \geq 1$, we conclude that the estimator of β is downward biased.

Appendix C Additional Simulation Results

The simulations for the case of heterogeneous slope are based on $5,000$ Monte Carlo replications. The results are summarized in Tables C.1, C.2, C.3, and C.4.

We also study the impact of missing values on our results. Here we focus on DGP 3 with $N = 50$ and $T = 100$, examining the homogeneous slope case, and reporting Bias, RMSE, and Size in Table C.5.

Table C.1: Bias and RMSE of estimators with heterogeneous slope: $N = 50$, $T = 100$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Bias	-0.0000	-0.0000	-0.0000	-0.0000	-0.0011	-0.0000
	RMSE	0.0111	0.0130	0.0112	0.0130	0.3017	0.0111
DGP 2	Bias	0.0000	0.3854	-0.0001	0.3916	1.8616	-0.0000
	RMSE	0.0353	0.3918	0.0354	0.3983	1.8698	0.0353
DGP 3	Bias	0.0008	0.4519	0.0009	0.4567	0.9838	0.0008
	RMSE	0.0350	0.4562	0.0352	0.4608	0.9886	0.0350
DGP 4	Bias	0.0002	0.0002	0.0001	0.0002	0.0053	0.0002
	RMSE	0.0343	0.0357	0.0348	0.0360	0.4925	0.0343
DGP 5	Bias	-0.0005	0.1930	-0.0006	0.1960	0.9304	-0.0005
	RMSE	0.0347	0.1984	0.0347	0.2014	0.9348	0.0347
DGP 6	Bias	-0.0000	0.2262	0.0001	0.2283	0.4922	-0.0001
	RMSE	0.0349	0.2299	0.0351	0.2321	0.4952	0.0350
DGP 7	Bias	-0.0001	-0.0001	-0.0003	-0.0003	0.0042	-0.0000
	RMSE	0.0360	0.0359	0.0359	0.0358	0.1439	0.0350
DGP 8	Bias	0.0007	0.0007	0.0007	0.0009	0.0016	0.0007
	RMSE	0.0347	0.0343	0.0346	0.0356	0.0475	0.0345
DGP 9	Bias	-0.0007	-0.0005	-0.0005	-0.0005	-0.0002	-0.0006
	RMSE	0.035	0.0329	0.0352	0.0360	0.0361	0.0350
DGP 10	Bias	-0.0008	-0.0008	-0.0006	-0.0007	-0.0000	-0.0007
	RMSE	0.0357	0.0355	0.0360	0.0358	0.1437	0.0354
DGP 11	Bias	-0.0008	-0.0009	-0.0009	-0.0007	-0.0004	-0.0009
	RMSE	0.0347	0.0333	0.0349	0.0340	0.0468	0.0347
DGP 12	Bias	-0.0017	-0.0015	-0.0018	-0.0019	-0.0016	-0.0017
	RMSE	0.0348	0.0315	0.0349	0.0336	0.0358	0.0348

Note: This table shows the bias and root mean squared error (RMSE) based on $5,000$ replications for the heterogeneous case. CCEMG is the common correlated effects mean group estimator defined in equation (9), INF is the infeasible CCEMG estimator by assuming that the common factor is known, and MG is the mean group estimator by first running separate regressions for each individual by ignoring the common factor, and then taking the mean of the coefficients obtained from the first step. CCEP is the common correlated effects pooled estimator defined in equation (10). POOL is the pooled OLS estimator without controlling for the panel structure and cross-sectional correlation. AGG is the OLS estimator from the aggregated time series regression as defined in equation (12).

Table C.2: Size and power of estimators with heterogeneous slope: $N = 50$, $T = 100$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Size	0.0606	0.0576	0.0610	0.0530	0.1344	0.0614
	Power	0.3064	0.2348	0.3110	0.2376	0.1326	0.3086
DGP 2	Size	0.0600	1.0000	0.0598	1.0000	1.0000	0.0598
	Power	0.3162	1.0000	0.3182	1.0000	1.0000	0.3146
DGP 3	Size	0.0614	1.0000	0.0620	1.0000	1.0000	0.0618
	Power	0.3192	1.0000	0.3240	1.0000	1.0000	0.3226
DGP 4	Size	0.0532	0.0504	0.0548	0.0530	0.1342	0.0530
	Power	0.3296	0.2986	0.3246	0.2964	0.1502	0.3290
DGP 5	Size	0.0572	0.9974	0.0590	0.9970	1.0000	0.0572
	Power	0.3100	1.0000	0.3108	1.0000	1.0000	0.3084
DGP 6	Size	0.0642	1.0000	0.0614	1.0000	1.0000	0.0630
	Power	0.3218	1.0000	0.3162	1.0000	1.0000	0.3218
DGP 7	Size	0.0596	0.0582	0.0596	0.0588	0.1650	0.0598
	Power	0.3036	0.2982	0.3070	0.2968	0.1932	0.3152
DGP 8	Size	0.0592	0.0230	0.0584	0.0224	0.2508	0.0558
	Power	0.3212	0.1848	0.3138	0.1784	0.4942	0.3222
DGP 9	Size	0.064	0.0146	0.0574	0.0184	0.4436	0.0632
	Power	0.3154	0.1518	0.3198	0.1518	0.7474	0.3124
DGP 10	Size	0.0682	0.0596	0.0686	0.0622	0.1644	0.0686
	Power	0.3178	0.3050	0.3122	0.3024	0.1790	0.3204
DGP 11	Size	0.0594	0.0432	0.0642	0.0458	0.2464	0.0574
	Power	0.3206	0.2896	0.3200	0.2888	0.4810	0.3180
DGP 12	Size	0.0592	0.0422	0.0568	0.0420	0.4398	0.0564
	Power	0.3114	0.3038	0.3144	0.2776	0.7484	0.3146

Note: This table reports the proportion of rejections of the null hypothesis that $\beta = 0$, when $\beta = 0$ (for size of various estimators) or $\beta = 0.05$ (for power of various estimators), based on $5,000$ replications for the heterogeneous case. For the description of different estimators, see the note to Table C.1.

Table C.3: Bias and RMSE of estimators with heterogeneous slope: $N = 100$, $T = 50$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Bias	0.0000	0.0010	-0.0000	0.0008	0.0792	0.0000
	RMSE	0.0287	0.0363	0.0283	0.0351	1.8911	0.0287
DGP 2	Bias	-0.0003	0.3969	-0.0003	0.3961	1.9299	-0.0003
	RMSE	0.0286	0.4061	0.0282	0.4057	1.9349	0.0288
DGP 3	Bias	0.0003	0.4577	0.0003	0.4557	0.9914	0.0004
	RMSE	0.0285	0.4629	0.0286	0.4602	0.9940	0.0286
DGP 4	Bias	0.0004	0.0008	0.0005	0.0009	0.0346	0.0003
	RMSE	0.0286	0.0306	0.0284	0.0302	0.9572	0.0287
DGP 5	Bias	-0.0005	0.1978	-0.0004	0.1974	0.9635	-0.0005
	RMSE	0.0288	0.2034	0.0285	0.2032	0.9665	0.0289
DGP 6	Bias	0.0003	0.2292	0.0003	0.2280	0.4963	0.0002
	RMSE	0.0283	0.2326	0.0281	0.2312	0.4981	0.0285
DGP 7	Bias	0.0004	0.0004	0.0004	0.0004	0.0015	0.0003
	RMSE	0.0299	0.0296	0.0293	0.0292	0.1959	0.0287
DGP 8	Bias	-0.0005	-0.0003	-0.0005	-0.0004	0.0003	-0.0005
	RMSE	0.0287	0.0280	0.0284	0.0284	0.0418	0.0287
DGP 9	Bias	-0.0002	-0.0000	-0.0003	-0.0003	-0.0000	-0.0001
	RMSE	0.0288	0.0255	0.0286	0.0274	0.0287	0.0289
DGP 10	Bias	-0.0004	-0.0004	-0.0004	-0.0004	-0.0023	-0.0004
	RMSE	0.0290	0.0287	0.0285	0.0284	0.1940	0.0288
DGP 11	Bias	0.0005	0.0006	0.0005	0.0005	0.0014	0.0006
	RMSE	0.0288	0.0268	0.0286	0.0271	0.0428	0.0289
DGP 12	Bias	0.0001	0.0001	-0.0001	0.0002	0.0005	0.0001
	RMSE	0.0289	0.0249	0.0286	0.0261	0.0292	0.0290

Note: This table shows the bias and root mean squared error (RMSE) based on $5,000$ replications for the heterogeneous case. For the description of different estimators, see the note to Table C.1.

Table C.4: Size and power of estimators with heterogeneous slope: $N = 100$, $T = 50$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
DGP 1	Size	0.0546	0.0534	0.0558	0.0530	0.1280	0.0570
	Power	0.4222	0.3004	0.4302	0.3086	0.1252	0.4212
DGP 2	Size	0.0546	1.0000	0.0542	1.0000	1.0000	0.0542
	Power	0.4208	1.0000	0.4354	1.0000	1.0000	0.4154
DGP 3	Size	0.0548	1.0000	0.0590	1.0000	1.0000	0.0542
	Power	0.4248	1.0000	0.4286	1.0000	1.0000	0.4200
DGP 4	Size	0.0532	0.0550	0.0532	0.0528	0.1314	0.0534
	Power	0.4260	0.3842	0.4342	0.3888	0.1318	0.4240
DGP 5	Size	0.0518	0.9996	0.0516	0.9992	1.0000	0.0530
	Power	0.4232	1.0000	0.4338	1.0000	1.0000	0.4234
DGP 6	Size	0.0510	1.0000	0.0548	1.0000	1.0000	0.0510
	Power	0.4308	1.0000	0.4422	1.0000	1.0000	0.4320
DGP 7	Size	0.0522	0.0538	0.0564	0.0554	0.1418	0.0530
	Power	0.3846	0.3842	0.4026	0.3976	0.1600	0.4168
DGP 8	Size	0.0574	0.0234	0.0530	0.0250	0.1930	0.0568
	Power	0.4110	0.2910	0.4256	0.2898	0.4686	0.4088
DGP 9	Size	0.0544	0.0138	0.0616	0.0166	0.3314	0.0574
	Power	0.4284	0.2990	0.4306	0.2772	0.7882	0.4250
DGP 10	Size	0.0558	0.0538	0.0556	0.0510	0.1402	0.0582
	Power	0.4118	0.4080	0.4266	0.4150	0.1552	0.4250
DGP 11	Size	0.0550	0.0396	0.0536	0.0422	0.1988	0.0540
	Power	0.4248	0.4278	0.4322	0.4184	0.4658	0.4198
DGP 12	Size	0.0530	0.0416	0.0536	0.0412	0.3418	0.0522
	Power	0.4188	0.4738	0.4330	0.4458	0.7798	0.4196

Note: This table reports the proportion of rejections of the null hypothesis that $\beta = 0$, when $\beta = 0$ (for size of various estimators) or $\beta = 0.05$ (for power of various estimators), based on $5,000$ replications for the heterogeneous case. For the description of different estimators, see the note to Table C.1.

Table C.5: Bias, RMSE, and Size of estimators with homogeneous slope: $N = 50$, $T = 100$

	Measure	CCEMG	MG	CCEP	POOL	AGG	INF
	Bias	-0.0002	0.4835	-0.0001	0.4816	0.9824	-0.0001
$p = .10$	RMSE	0.0193	0.4868	0.0186	0.4845	0.9866	0.0193
	Size	0.0540	1.0000	0.0556	1.0000	1.0000	0.0536
	Bias	-0.0002	0.5161	-0.0002	0.5093	0.9774	-0.0001
$p = .20$	RMSE	0.0179	0.5193	0.0172	0.5121	0.9817	0.0180
	Size	0.0494	1.0000	0.0464	1.0000	1.0000	0.0516
	Bias	-0.0003	0.5527	-0.0002	0.5414	0.9728	-0.0004
$p = .30$	RMSE	0.0175	0.5557	0.0168	0.5442	0.9769	0.0175
	Size	0.0512	1.0000	0.0522	1.0000	1.0000	0.0506
	Bias	-0.0002	0.5890	-0.0002	0.5749	0.9628	-0.0002
$p = .40$	RMSE	0.0167	0.5919	0.0159	0.5776	0.9670	0.0168
	Size	0.0506	1.0000	0.0472	1.0000	1.0000	0.0486
	Bias	-0.0002	0.6288	-0.0002	0.6131	0.9513	-0.0002
$p = .50$	RMSE	0.0165	0.6319	0.0158	0.6160	0.9557	0.0166
	Size	0.0554	1.0000	0.0562	1.0000	1.0000	0.0552
	Bias	-0.0001	0.6665	-0.0002	0.6514	0.9332	-0.0002
$p = .60$	RMSE	0.0165	0.6696	0.0157	0.6544	0.9376	0.0165
	Size	0.0572	1.0000	0.0584	1.0000	1.0000	0.0590
	Bias	-0.0002	0.7012	-0.0004	0.6888	0.9018	-0.0002
$p = .70$	RMSE	0.0163	0.7045	0.0154	0.6920	0.9063	0.0164
	Size	0.0616	1.0000	0.0626	1.0000	1.0000	0.0604
	Bias	-0.0003	0.7153	-0.0002	0.7075	0.8403	-0.0003
$p = .80$	RMSE	0.0170	0.7196	0.0154	0.7117	0.8455	0.0171
	Size	0.0592	1.0000	0.0630	1.0000	1.0000	0.0598

Note: This table shows the bias, root mean squared error (RMSE), and size, based on $5,000$ replications for the homogeneous case, where a proportion p of both the dependent and independent variables are missing at random. CCEMG is the common correlated effects mean group estimator defined in equation (9), INF is the infeasible CCEMG estimator by assuming that the common factor is known, and MG is the mean group estimator by first running separate regressions for each individual by ignoring the common factor, and then taking the mean of the coefficients obtained from the first step. CCEP is the common correlated effects pooled estimator defined in equation (10). POOL is the pooled OLS estimator without controlling for the panel structure and cross-sectional correlation. AGG is the OLS estimator from the aggregated time series regression as defined in equation (12).