Health Shocks in a General Equilibrium Model*

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Abstract
We analyze the impact of health shocks in a general equilibrium model with two sectors: one sector being less susceptible to the pandemic and the other sector being more sensitive to the pandemic. The health shock leads to a reduction in labor supply, utilization of labor and demand for products of firms in the pandemic-sensitive sector, and a decrease in the willingness of consumers to spend on the products of both sectors. While health shocks have supply-side effects on economic activity, the demand-side effects are considerably bigger for shorter horizons and more rigid prices. With dominant demand-side effects, aggregate inflation tends to decrease during the early COVID-19 crisis. Finally, another important channel through which the health shock affects economic activity is the reallocation of resources from the more susceptible sector to the less susceptible one.

JEL classification: E31; E32
Keywords: COVID-19, DSGE, Health Shocks, Labor Hoarding, Reallocation, Price Rigidity

*We thank Sanjay K. Chugh, Nicolas Castro Cienfuegos and Steve Davis for helpful discussions.
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1 Introduction

In light of the outbreak of the COVID-19 pandemic, there is an ongoing interest in studying the effects of pandemics on the macroeconomy. An important question following the outbreak is the nature of the shock and the way that it propagates in the economy. This paper studies the implications of a health shock in a dynamic stochastic general equilibrium (DSGE) model in which one sector is particularly sensitive to pandemics. We are interested in quantifying the relative importance of the demand-side and the supply-side effects of a health shock.

In the benchmark model, one sector (to which we refer as Sector 1) is less susceptible to the pandemic, while the other sector (Sector 2) is more sensitive to pandemics. There are four potential channels through which the health shock affects economic activity. First, a deterioration in the state of public health reduces the desire to work in the pandemic-sensitive sector. Second, consumers reduce the demand for the products of the pandemic-sensitive sector. This not only reflects a decline in the labor input, but a decline in the desire to consume these products. This is a direct demand effect that the pandemic has on Sector 2 that is distinct from the labor supply effect. Third, given the pandemic-induced uncertainty, households may choose to delay consumption of the products of both sectors. We model this effect through a direct impact of the pandemic on the patience rates of households. The fourth effect is a potential adjustment of labor utilization (“labor hoarding”) in the pandemic-sensitive sector that minimizes the impact on labor in this sector.

The health shock manifests itself as both demand and supply shocks. While viewing the COVID-19 pandemic as a shock to labor supply has merits, the demand effect is also very significant. Indeed, for shorter horizons (within 2-4 quarters in our benchmark analysis), the demand effect on labor and output dominates the supply effect by nearly a 2:1 margin. For longer horizons, the supply effect dominates, but the demand effect remains meaningful, particularly in the pandemic-sensitive sector. When the direct labor supply effect is turned off, the negative impact on output and labor is reduced by roughly 30-40%. When the demand effect is turned off, however, the negative impact is reduced by at least 60%.

Price rigidity is the key to understanding the relative strength of each channel and its impact on the economy. With fully flexible prices in Sector 2, the supply-side effect dominates. When prices are fully flexible in both sectors, only the supply-side effect is operative. On the other hand, with fully rigid prices (whether in Sector 2 only or both sectors), the demand-side effect is the sole channel through which the health shock impacts economic activity. For intermediate values of price rigidity, both effects are important, with the demand effect being stronger for more rigid prices. Intuitively, with price rigidity, output is demand determined, and absent any demand-side effects from Sector 2, the health shock does not alter labor and output.

The effect of the health shock on inflation in each sector depends on the scenario at hand. With only a supply-side effect, inflation in the pandemic-sensitive sector rises. With only a demand-side effect, the opposite occurs. With both effects, inflation in Sector 2 is mostly unchanged. The behavior of aggregate inflation follows that of the pandemic-sensitive sector. For aggregate inflation to fall, we would expect to see a considerably stronger demand-side than a supply-side effect. In
March and April 2020, the Consumer Price Index for All Urban Consumers (CPI-U) declined by 0.4% and 0.8%, respectively. This observation is consistent with our finding on the relative strength of the demand-side effect of the health shock.

Our theoretical model is motivated by the empirical literature on disentangling agents’ perceptions of the COVID-19 pandemic. Some studies argue that the pandemic is a demand shock, and generally rely upon a direct approach to uncovering the perceptions of firms regarding the nature of the pandemic through surveys and first source evidence. Hassan et al. (2020) analyze transcripts of quarterly earnings calls held by public firms and find concerns over a negative demand shock are nearly twice as prevalent as mentions of supply chain disruptions. In a survey of small firms, Bartik et al. (2020) find respondents cite reductions in demand to a much larger degree than supply chain issues as reasons for temporary closures. Meyer et al. (2020) show that firms within the sixth Federal Reserve district, on net, view this crisis largely as a demand rather than a supply shock, since firms lower wages for a material share of their workforce and anticipate further wage cuts and lower prices over the near term. Balleer et al. (2020) find strikingly similar results in a survey of German firms. Others argue that the pandemic is largely a supply shock. Brinca et al. (2020) decompose changes in hours working into supply and demand shock contributions, finding that the supply shock contribution outweighs that of demand. Candia et al. (2020) suggest that households and some firms see the pandemic as a supply shock through aggregate inflation expectations. Dietrich et al. (2020) reach the same conclusion through household surveys that elicit expectations for the COVID-19 pandemic’s impact on aggregate inflation.

Our paper adds to the burgeoning literature inspired by the outbreak of COVID-19; see, e.g. Eichenbaum et al. (2020), Kruger et al. (2020), Bodenstein et al. (2020) and McKibbin and Fernando (2020), among many others.\textsuperscript{1} Our work differs from these studies by quantifying the relative strength of the demand and supply effects of the health shock on the economy. In this regard, our paper is most closely related to Guerrieri et al. (2020) in which shock to supply could lead to deficient demand and the drop in demand could be bigger than the shock itself. Low substitutability across sectors and incomplete markets contribute to the possibility of generating Keynesian supply shocks. The key to their findings is complementarity between both sectors; a decline in activity in one sector reduces activity in the other. In contrast, labor and output of Sector 1 in our model are not directly affected by a pandemic, but they could fall even if the products of both sectors are substitutes. This occurs because a decline in labor in the pandemic-sensitive sector reduces total labor income, and thus reduces demand for the products of both sectors, regardless of the assumption about substitutability.

Finally, our work is related to empirical studies on health shocks at the individual level, e.g.

\textsuperscript{1}Eichenbaum et al. (2020) demonstrate that cutting consumption and work reduces the severity of the epidemic (as measured by total deaths), but exacerbates the recession caused by the epidemic. Kruger et al. (2020) argue that endogenous shifts in private consumption behavior across sectors can be a mitigating mechanism during an epidemic or when the economy is re-opened after a lockdown. Bodenstein et al. (2020) discuss the effects of social distancing on the supply chain using a two-sector model that features a core sector that produces intermediate inputs that are not easily replaced by inputs from the other sector. McKibbin and Fernando (2020) consider seven different scenarios that are related to the outbreak of COVID-19 and conclude that even a contained outbreak could have a significant impact on the global economy in the short run.
Bradley et al. (2012), Mohanan (2013) and Sinclair and Smetters (2004). Our paper differs from this line of research in that we address the impact of shocks to public health, not necessarily to an individual’s health, on the macroeconomy.

The remainder of the paper is organized as follows. Section 2 outlines the benchmark model with the health shock. Analytical analyses are presented in Section 3. Section 4 describes the calibration of the model and presents numerical results. Section 5 provides robustness analyses and presents a three-sector model that allows for reallocation across sectors. Section 6 concludes.

2 The Model

The economy is populated by a continuum of infinitely-lived households who derive utility from consumption from Sector 1 and Sector 2, and supply labor services to both services. Firms in both sectors hire labor as the only input to produce differentiated products and face adjustment cost functions for prices. Sector 1 makes products that are less susceptible to pandemics while Sector 2 makes products or provides services that are sensitive to pandemics, such as the hospitality industry, sports, restaurants, and other venues with large public gatherings.

The supply of labor to Sector 2 is largely affected by the threat of the spread of a pandemic, and thus some workers may choose not to work during this episode. Similarly, due to the potential spread of the pandemic and social distancing, the demand for the products (or services) of Sector 2 are reduced. During a pandemic, households may reduce their demand for the products of both sectors as they may tilt towards more saving (i.e. become more patient). Our baseline model allows firms in both sectors to adjust their labor demand in the face of a negative shock to public health. We also explore the possibility that firms in the hard-hit sector may resort to “labor hoarding” by changing the utilization of labor.

2.1 Households

In each period $t$, a representative household derives utility from consumption of Sector 1 ($c_{1,t}$) and Sector 2 ($c_{2,t}$), supplies labor to both sectors ($n_{1,t}$ and $n_{2,t}$) as well as holds bonds ($B_t$). In addition, in Sector 2, the utilization of labor is given by $u_{2,t}$, and it is determined by firms as we discuss later. The problem of the representative household is then given by:

$$\max_{\{B_t,c_{1,t},c_{2,t},n_{1,t},n_{2,t}\}_{t=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^i \zeta_t \left( \frac{c_{1,t}^{1-\sigma_1}}{1-\sigma_1} + \phi_t \frac{c_{2,t}^{1-\sigma_2}}{1-\sigma_2} - \chi_t \frac{n_{1,t}^{1+\nu_1}}{1+\nu_1} - \gamma_t \frac{(u_{2,t}^{1+\nu_2})^{1+\nu_2}}{1+\nu_2} \right)$$

(1)

with $\beta < 1$ being the household’s subjective discount factor, $\zeta_t$ is a taste shifter that captures changes in the patience rate, $\sigma_1 > 1$ and $\sigma_2 > 1$ are the curvature parameters of consumption, $\nu_1 > 0$ and $\nu_2 > 0$ are the inverse of the labor supply elasticities in each sector, and $E_t$ is the expectations operator. The variables $\chi_t$, $\phi_t$ and $\gamma_t$ measure the relative weights in the utility function that are attached to consumption of Sector 1 products, labor in Sector 1 and labor in Sector 2, respectively.

Consumption in each sector $i$ is a Dixit-Stiglitz composite of differentiated products that are
produced by a continuum of final-good firms (indexed by \(j\)) in each sector:

\[ c_{i,t} = \left( \int_0^1 \frac{c_{i,j,t}}{P_{i,j,t}} dj \right)^{\frac{\varepsilon_i}{1-\varepsilon_i}} \]  

(2)

with \(\varepsilon_i\) being the elasticity of substitution between product varieties in each sector, and \(i = 1, 2\). The corresponding aggregate price in each sector is then given by

\[ P_{i,t} = \left( \int_0^1 P_{i,j,t}^{1-\varepsilon_i} dj \right)^{-\frac{1}{\varepsilon_i}} \]

In addition, the optimal allocation over the differentiated varieties yield:

\[ c_{i,j,t} = \left( \frac{P_{i,j,t}}{P_{i,t}} \right)^{-\varepsilon_i} c_{i,t} \]  

(3)

Maximization is subject to the sequence of budget constraints:

\[ P_{1,t}c_{1,t} + P_{2,t}c_{2,t} + B_t = W_{1,t}n_{1,t} + W_{2,t}n_{2,t}u_{2,t} + R_{t-1}B_{t-1} + P_{1,t}T_t + P_{1,t}\Pi_t \]  

(4)

where \(P_{1,t}\) is the price of \(c_{1,t}\), \(P_{2,t}\) is the price of \(c_{2,t}\), \(W_{1,t}\) is the nominal wage in Sector 1, \(W_{2,t}\) is the nominal wage in Sector 2, \(T_t\) are net transfers, \(\Pi_t\) are profits and \(R_t\) is the gross nominal interest rate on bonds. Dividing both sides of (4) by \(P_{1,t}\), the budget constraint in real terms reads:

\[ c_{1,t} + p_{2,t}c_{2,t} + b_t = w_{1,t}n_{1,t} + w_{2,t}n_{2,t}u_{2,t} + \frac{R_{t-1}b_{t-1}}{\pi_{1,t}} + T_t + \Pi_t \]  

(5)

with \(p_{2,t}\) is the relative price of \(c_{2,t}\), \(\pi_{1,t}\) is the gross inflation rate in Sector 1, and lower-case variables correspond to the real value of the upper-case values.

Denoting the Lagrange multiplier on equation (5) by \(\lambda_t\), the optimal choices of consumption, labor and bond holdings yield the following conditions:

\[ c_{1,t}^{-\sigma_1} = \lambda_t \]  

(6)

\[ \phi_t c_{2,t}^{-\sigma_2} = \lambda_t p_{2,t} \]  

(7)

\[ \chi_t n_{1,t} = \lambda_t w_{1,t} \]  

(8)

\[ \gamma_t (u_{2,t}n_{2,t})^{\nu_2} = \lambda_t w_{2,t} \]  

(9)

\[ \lambda_t \zeta_t = \beta R_t \mathbb{E}_t \left( \frac{\zeta_{t+1} \lambda_{t+1}}{\pi_{1,t+1}} \right) \]  

(10)

Combining conditions (6) and (10) gives the Euler equation for the consumption of Sector-1 products, and combining conditions (7) and (10) gives the Euler equation for the consumption of Sector-2 products. Similarly, combining conditions (6) and (8) gives the labor supply condition to the first sector and combining conditions (7) and (9) gives the labor supply condition to the second sector.

In our benchmark setup, we use a separable utility function. This specification is useful in that it allows us to conduct analytical analyses. In the robustness analyses, we show the results using a more general form of the utility function. Specifically, we present a model where both Sector
1 and Sector 2 consist of a wide range of sectors with higher within-sector substitutability than between-sector substitutability. Finally, in what follows, we focus on the effects of the health shock on Sector 2; for this reason, we let $\chi_t$ be constant at its steady-state value ($\chi$).

### 2.2 The Production Sector

The firms in both sectors are similar in every aspect with two exceptions. First, firms in Sector 2 may resort to changes in the utilization of labor while we assume that firms in Sector 1 do not. Second, productivity might differ across sectors. Our two-sector model allows for monopolistic competition in each sector so that, as discussed above, households can choose different varieties in the same sector. As such, we distinguish between within-sector substitution and between sector substitution. Firms also set their prices optimally and incur an adjustment cost as in Rotemberg (1982). Since most of model features are standard, we relegate the detailed analysis to Appendix A.1 and focus on the main optimality conditions of each sector.

#### 2.2.1 Sector 1

Each firm $j$ hires labor as the only input to produce a differentiated product using the following technology:

$$y_{1,j,t} = z_{1,t}n_{1,j,t}^{\alpha_1}$$

with $\alpha_1 < 1$ and $z_{1,t}$ being total factor productivity (TFP), which is common to all firms in this sector. Optimization with respect to labor and imposing symmetry gives the following labor demand condition:

$$\alpha_1 mc_{1,t}z_{1,t}n_{1,t}^{\alpha_1 - 1} = w_{1,t}.$$  

with $mc_{1,t}$ being the real marginal cost of each firm in Sector 1.

In a symmetric equilibrium, in which all firms set the same price, the optimal choice of prices leads to the following standard forward-looking Phillips curve:

$$1 - \eta_1 (\pi_{1,t} - 1)\pi_{1,t} + \beta \eta_1 \mathbb{E}_t \left\{ \frac{\lambda_{t+1}\zeta_{t+1}}{\lambda_t \zeta_t} (\pi_{1,t+1} - 1)\pi_{1,t+1} \frac{y_{1,t+1}}{y_{1,t}} \right\} = \varepsilon_1 (1 - \alpha_1 mc_{1,t}).$$

where $\eta_1$ is the parameter that governs the degree of price rigidity in Sector 2. In the case of fully flexible prices ($\eta_1 = 0$), equation (13) becomes $mc_{1,t} = \frac{\varepsilon_1 - 1}{\alpha_1 \varepsilon_1}$, which is the inverse of the steady-state gross price markup.

#### 2.2.2 Sector 2

Each firm $j$ hires labor as the only input to produce a differentiated product using the following technology:

$$y_{2,j,t} = z_{2,t}(u_{2,j,t}n_{2,j,t})^{\alpha_2}$$

1 and Sector 2 consist of a wide range of sectors with higher within-sector substitutability than between-sector substitutability. Finally, in what follows, we focus on the effects of the health shock on Sector 2; for this reason, we let $\chi_t$ be constant at its steady-state value ($\chi$).
with \( z_{2,t} \) being total factor productivity (which is common to all firms in this sector). Notice that, unlike in sector 1, we allow for the possibility of changes in the utilization of labor (\( u_{2,j,t} \)) as firms in this hard-hit sector may choose to change their utilization of the labor input rather than the level of labor itself. Changing utilization, however, entails the following resource cost:

\[
\Psi_{2,j,t} = \frac{\psi}{2} \left( \frac{u_{2,j,t}}{u_{2,j,t-1}} - 1 \right)^2 y_{2,t}
\]

After taking first-order conditions and imposing symmetry across firms in this sector, the labor demand condition, the choice of labor utilization and the Phillips curve are, respectively, given by:

\[
\alpha_2 m_{2,t} z_{2,t} (u_{2,t} n_{2,t})^{\alpha_2 - 1} = w_{2,t}
\]

\[
\alpha_1 m_{2,t} z_{2,t} (u_{2,t} n_{2,t})^{\alpha_2 - 1} u_{2,t}^{-1} = w_{2,t} + \Psi_{2,t} z_{2,t} (u_{2,t} n_{2,t})^{\alpha_2 - 1}
\]

\[
1 - \eta_2 (\pi_{2,t} - 1) \pi_{2,t} + \beta \eta_2 \mathbb{E}_t \left[ \frac{\lambda_{t+1} \zeta_{t+1}}{\lambda_t \zeta_t} (\pi_{2,t+1} - 1) \pi_{2,t+1} \frac{y_{2,t+1}}{y_{2,t}} \right] = \varepsilon_2 (1 - \alpha_2 m_{2,t})
\]

where \( m_{2,t} \) is the real marginal cost and:

\[
\Psi_{2,t}' = \psi \left( \frac{u_{2,t}}{u_{2,t-1}} - 1 \right) \frac{y_{2,t}}{y_{2,t-1}} - \beta \psi \left[ \frac{\lambda_{t+1} \zeta_{t+1}}{\lambda_t \zeta_t} \left( \frac{u_{2,t+1}}{u_{2,t}} - 1 \right) \frac{u_{2,t+1}}{u_{2,t}} \right].
\]

In addition, the relative price of Sector 2 evolves according to the following equation:

\[
\frac{p_{2,t}}{p_{1,t-1}} = \frac{\pi_{2,t}}{\pi_{1,t-1}}
\]

which ties the path of the relative price of Sector 2 to the ratio of inflation in both sectors.

### 2.3 The Health Shock

We assume that the state of public health \( H_t \) is given by:

\[
\ln \left( \frac{H_t}{\overline{H}} \right) = \rho_H \ln \left( \frac{H_{t-1}}{\overline{H}} \right) + \iota_{H,t}
\]

with \( \overline{H} = 1 \) being the steady-state value of \( H_t \), \( \rho_H \) is the persistence of the health shock, and \( \iota_{H,t} \sim \mathcal{N}(0, \sigma_{H}^2) \). We allow for persistence in the health shock such that the shock only gradually fades. In addition, we assume that the representative household takes the state of public health as given.\(^2\)

The supply of labor to Sector 2 is affected by the state of public health: \( \gamma_t = \Gamma(H_t) \), with \( \Gamma'(H_t) < 0 \). As such, a negative health shock raises the disutility of working in Sector 2 and reduces the supply of labor to this sector. Similarly, consumption in Sector 2 is positively related to the

\(^2\)Later in the paper, we allow the state of public health to be endogenous in response to the reduced activity in Sector 2, and find very similar results; see Section 5.6.
state of public health: $\phi_t = \Phi(H_t)$, with $\Phi'(H_t) > 0$. A negative health shock, thus, reduces the demand for consumption in Sector 2.

We now describe the derivations of the functions $\Phi(H_t)$ and $\Gamma(H_t)$. One way to view the variables $\gamma_t$ and $\phi_t$ is that they result from some complementarity between the state of health, on one hand, and labor and consumption in Sector 2, on the other hand. As shown in Appendix A.2, we obtain the following equations that describe the evolutions of $\gamma_t$ and $\phi_t$:

$$\gamma_t = \gamma H_t^{m_2(1+\nu_2)} \tag{22}$$

$$\phi_t = \phi H_t^{d_2(1-\sigma_2)} \tag{23}$$

with $\gamma$ and $\phi$ being the steady-state values of the corresponding variables, and they will be determined in the numerical analysis section. In addition, $m_2 < 0$ and $d_2 < 0$ govern the complementarity between the state of health and consumption/labor in Sector 2. With this characterization, the marginal utility of consumption is increasing in the state of public health, which is consistent with the empirical findings of Finkelstein et al. (2013).

### 2.4 The Reallocation Effects of a Health Shock

One implication of these assumptions is that the state of public health not only affects consumption of Sector 2's products, but also has implications for the relative consumption levels. To see this, combine conditions (6)-(7) and $\Phi_t$ to obtain:

$$\frac{c^\sigma_{2,t}}{c^\sigma_{1,t}} = \frac{\Phi(H_t)}{p_{2,t}} \tag{24}$$

An increase in the relative price $c^\sigma_{2,t}$ or a decline in the state of public health reduces the demand for $c^\sigma_{2,t}$ relative to $c^\sigma_{1,t}$, implying a reallocation between sectors that respond differently to the deterioration in the state of public health. This reallocation may happen with consumption of both sectors falling, but may also reflect a decline in the demand for Sector-2 products with an increase in the demand for Sector-1 products. In this respect, the response of the relative price $p_{2,t}$ is also important.

The relative shift towards $c_{1,t}$ also implies a relative shift to supplying labor to Sector 1. In this regard, the combination of conditions (8)-(9) and $\Gamma_t$ gives:

$$\frac{n^\nu_{2,t}}{n^\nu_{1,t}} = \frac{\Gamma(H_t)w_{2,t}}{\Gamma(H_t)w_{1,t}} \tag{25}$$

As such, a health shock shifts resources across sectors. Notice also that the shift away of $n_{2,t}$ is affected by the behavior of labor utilization; a decline in the utilization of labor prevents a bigger decline in labor.
2.5 Market Clearing, Monetary Policy and the Taste Shifter

In equilibrium, bonds are in zero net supply \((b_t = 0)\). In addition, Sector 1 and Sector 2 clear:

\[
z_{1,t} \alpha_{1,t} \left[ 1 - \frac{\eta_1}{2} (\pi_{1,t} - 1)^2 \right] = c_{1,t} \quad (26)
\]

\[
z_{2,t} (u_{2,t} n_{2,t})^\alpha_2 \left[ 1 - \frac{\eta_2}{2} (\pi_{2,t} - 1)^2 - \frac{\psi}{2} \left( \frac{u_{2,t}}{u_{2,t-1}} - 1 \right)^2 \right] = c_{2,t} \quad (27)
\]

Monetary policy is governed by a Taylor-type rule with interest rate smoothing whereby the nominal interest rate responds to deviations of inflation from its steady-state value as follows:

\[
\ln \left( \frac{R_t}{R} \right) = \rho_R \ln \left( \frac{R_{t-1}}{R} \right) + (1 - \rho_R) \rho_\pi \ln \left( \frac{\pi_t}{\bar{\pi}} \right) \quad (28)
\]

with \(\pi_t\) being aggregate inflation, \(\bar{\pi}\) its steady-state value, \(\rho_\pi > 1\) and \(\rho_R > 0\) being the coefficients of inflation and interest rate smoothing, respectively. Aggregate inflation is a geometric average of inflation rates of both sectors, with the weights being determined by the share of output of each sector in total output; namely \(\pi_t = \pi_{1,t} s_t \pi_{2,t}^{1-s_t}\), and \(s_t\) being the time-varying share of Sector 1. We also note that the use of the interest rate is solely to close the model (i.e. to determine \(R_t\)) and that the behavior of monetary policy in the face of pandemics is not the main goal of this paper.

Finally, the taste shifter evolves according to the following rule:

\[
\zeta_t = \overline{\zeta} \Pi_t^q \quad (29)
\]

so that the dynamics for this variable is similar to the dynamics for \(\phi_t\) and \(\gamma_t\). For the patience rate of the households to rise following a negative health shock, we have \(q < 0\). In addition, \(\overline{\zeta} = 1\).

3 Analytical Analysis

The role of price rigidity and the importance of the demand and supply effects are illustrated in this section. For simplicity, we assume a fixed discount factor \((\zeta_t = 1 \text{ for all } t)\), no interest-rate smoothing \((\rho_R = 0)\) and, unless otherwise stated, full labor utilization \((u_t = 1 \text{ for all } t)\). We also note that the main conclusions of this section apply both for a time-varying interest rate \((R_t \neq 0)\) and a constant interest rate \((R_t = 0)\).

3.1 Flexible Prices

Combining conditions (6) and (8) and log-linearizing around the deterministic steady state gives:

\[
\nu_1 \overline{\pi}_{1,t} + \sigma_1 \overline{c}_{1,t} = \overline{\omega}_{1,t} \quad (30)
\]

with \(\overline{\pi}_t\) being the log deviation of any variable \(x_t\) from its non-stochastic steady state \((\overline{x})\).

With flexible prices, the marginal cost is constant; therefore, \(\overline{\omega}_{1,t} = (\alpha_1 - 1) \overline{\pi}_{1,t}\). The log-linearized
version of the production function of sector 1 is given by \( y_{1,t} = \alpha_1 \bar{n}_{1,t} \), and market clearing gives \( \bar{c}_{1,t} = \bar{y}_{1,t} \). Then, condition (30) yields \( \bar{y}_{1,t} = 0 \). Labor, consumption and output of Sector 1, thus, are irresponsive to the health shock.

Similar analyses in Sector 2 give:

\[
\left( \frac{1 + \nu_2 - \alpha_2}{\alpha_2} \right) \bar{y}_{2,t} = -\bar{\gamma}_t
\]  

(31)

Using condition (31) and the log-linearized version of condition (22), we obtain:

\[
\frac{\bar{y}_{2,t}}{\bar{h}_t} = \frac{-m_2 (1 + \nu_2) \alpha_2}{1 + \nu_2 - \alpha_2} > 0
\]  

(32)

A decline in the state of public health reduces output of the pandemic-sensitive sector. In deriving the response of Sector 2 to the health shock, we only use the labor supply condition. Put differently, we only make use of the effect of the health shock on the disutility of labor \( \bar{\gamma}_t \), but not the direct effect on the demand from Sector 2 \( \bar{\phi}_t \). Therefore, in a fully flexible-price setting, the direct demand-side effect becomes irrelevant to the dynamics of total labor and output.\(^3\)

### 3.2 Sticky Prices

The combination of equations (6) and (10) gives the Euler equation for Sector 1. After using the market clearing condition, the Euler equation in log deviations can be written as:

\[
\bar{R}_t - E_t \bar{\pi}_{1,t+1} = \sigma (E_t \bar{y}_{1,t+1} - \bar{y}_{1,t})
\]  

(33)

Since \( E_t \bar{x}_{t+1} = \rho_H \bar{x}_t \) and \( \bar{R}_t = \phi_H \bar{x}_{1,t} \), we have:

\[
(\phi_H - \rho_H) \bar{\pi}_{1,t} = \sigma (\rho_H - 1) \bar{y}_{1,t}
\]  

(34)

In Sector 2, the equivalent equation is given by:

\[
(\phi_H - \rho_H) \bar{\pi}_{1,t} = (\rho_H - 1)(\sigma_2 \bar{y}_{2,t} + \bar{p}_{2,t} - \bar{\phi}_t)
\]  

(35)

and the log-linearized Phillips curve:

\[
(1 - \beta \rho_H) \bar{\pi}_{2,t} = \kappa_2 \left( \frac{1 + \nu_2 - \alpha_2}{\alpha_2} \right) \bar{y}_{2,t} + \kappa_2 \bar{\gamma}_t
\]  

(36)

where \( \kappa_2 = \frac{\kappa - 1}{\eta_2} \) is the slope of the log-linearized Phillips curve. With sticky prices, the path of output in Sector 2 depends not only on the combination of the labor demand and labor supply conditions, but also on the behavior of \( \phi_t \) and the relative price \( p_{2,t} \). More generally, the demand side of the economy, as illustrated in conditions (34)-(35), is important too.

\(^3\)The result regarding Sector 1 reflects the assumption that \( \chi_t \) is constant. If \( \chi_t \) were allowed to vary, then Sector 1 would be impacted. However, this modification would not change the main conclusion of this subsection: with flexible prices, labor and output are determined only by the supply-side effect of the shock.
With fully sticky prices in both sectors \((\eta_i \to \infty), \kappa_2 = 0\) and \(\bar{\pi}_{1,t} = \bar{\pi}_{2,t} = 0\), which also implies a fixed relative price \((\bar{p}_{2,t} = 0)\). Then, in Sector 1, \(\bar{y}_{1,t} = 0\). In Sector 2, the Phillips curve, which summarizes the supply-side of the economy, is not informative (both sides of the equation equal zero). In particular, the direct effect of the health shock on the labor supply vanishes (even if \(\tilde{\gamma}_t\) is non-zero). On the other hand, equation (35), which summarizes the demand side of the economy, gives:

\[ \sigma_2 \bar{y}_{2,t} = \hat{\phi}_t \] (37)

suggesting that the evolution of output in Sector 2 is determined only by the direct demand-side effect of the health shock; the demand for Sector-2 products falls when \(\hat{\phi}_t < 0\).\(^4\)

Using equation (23), we can then relate the evolution in the output of Sector 2 to the health shock as follows:

\[ \frac{\bar{y}_{2,t}}{H_t} = \frac{-d_2 (\sigma_2 - 1)}{\sigma_2} \] (38)

which is positive provided that \(\sigma_2 > 1\) and \(d_2 < 0\), as assumed above.

Consider now the following case: prices in Sector 1 are rigid (but not fully rigid). As discussed above, if prices in Sector 2 are fully flexible, then output of Sector 2 is only affected by the supply-side channel. However, the demand-side channel affects \(\bar{\pi}_{1,t}\) (through equation (35)), which in turn affects output of Sector 1 (through condition (34)). Therefore, total output and labor in the economy are not solely determined by the supply-side effect. On the other hand, if prices of Sector 2 are fully rigid, then the supply-side effect vanishes, and output is determined only by the demand-side effect. We observe asymmetry regarding the importance of each channel: the supply-side effect is not operative when one sector has fully rigid prices, but the demand-side effect remains operative when one sector has fully flexible prices.

To summarize, with fully flexible prices in both sectors, only the supply-side effect of the health shock affects the economy. If prices are fully flexible only in Sector 2, then the demand-side effect remains important. When prices of both sectors are fully rigid, only the demand-side effect of the health shock is material. In any intermediate case, both effects matter. Price rigidity, thus, is crucial for the propagation of health shocks in the economy.

### 3.3 The Health Shock Multiplier

We briefly discuss the “health shock multiplier”, which is the change in output as a result of a given change in the state of public health. Consider flexible prices first. From equation (32), we can write this multiplier as:

\[ \frac{dy_{2,t}^f}{dH_t} = \frac{-m_2 (1 + \nu_2) \alpha_2 \bar{y}_2}{(1 + \nu_2 - \alpha_2) H} \] (39)

\(^4\)One may also combine the log deviations of conditions (6) and (7) to obtain condition (37), with \(\hat{\gamma}_{1,t} = 0\).
and it depends on the degree to which labor supply in Sector 2 responds to a health shock. We may also show the cumulative effect of a health shock over \( T \) periods ahead:

\[
\frac{\mathbb{E}_t \sum_{i=0}^T dy_{2,t+i}}{\mathbb{E}_t \sum_{i=0}^T dH_{t+i}} = \frac{-m_2 (1 + \nu_2) \alpha_2 y_2 (1 - \rho_H^T)}{(1 + \nu_2 - \alpha_2) H (1 - \rho_H)}
\]  

(40)

For fully sticky prices:

\[
\frac{dy_{2,t}}{dH_t} = \frac{-d_2 (\sigma_2 - 1) y_2}{\sigma_2 H}
\]  

(41)

\[
\frac{\mathbb{E}_t \sum_{i=0}^T dy_{2,t+i}}{\mathbb{E}_t \sum_{i=0}^T dH_{t+i}} = \frac{-d_2 (\sigma_2 - 1) y_2 (1 - \rho_H^T)}{\sigma_2 H (1 - \rho_H)}
\]  

(42)

We can now express the ratio of the effect of the health shock on output of Sector 2 with fully rigid prices to the effects with fully flexible prices as \( \frac{dy_{2,t}}{dH_t} / \frac{dy_{2,t}}{dH_t} \).

### 3.4 Changes in Labor Utilization

We now consider the possibility that firms in Sector 2 may choose to adjust labor utilization \((u_{2,t})\). With fully flexible prices, we obtain the following condition for Sector 2:

\[
\bar{n}_{2,t} = \frac{-\bar{\gamma}_t}{(1 + \nu_2 - \alpha_2)} - \bar{u}_{2,t}
\]  

(43)

which is a generalization of condition (43). In this case, labor of Sector 2 is not solely determined by the supply-side effect of the health shock, but also by the adjustment of utilization. If firms reduce their utilization \((\bar{u}_{2,t} < 0)\), then the drop in labor in Sector 2 will be smaller compared to a model in which the adjustment occurs only through changes to labor itself.

This modification, however, does not change the results with fully rigid prices; the path of output in Sector 2 continues to be determined by the demand-side effect as shown in condition (37). Since output is demand driven, whether firms in Sector 2 meet the reduced demand by either reducing labor, reducing utilization or both does not alter the ultimate path of output.

Finally, so far, we have assumed that the supply-side effect operates through labor supply. Alternatively, one could assume that productivity drops due to a pandemic (see, for example, Cespede et al. (2020)). In Appendix A.3, we show that our main results are unchanged if one introduces the supply-side effect through productivity (i.e. the firm side) rather than through labor supply.

### 4 Numerical Analysis

In Section 3, we analytically show that 1) with fully flexible prices in both sectors, only the supply-side effect of the health shock is operative on labor and output, 2) on the other extreme of fully rigid prices (in Sector 2 or in both sectors), only the demand-side effect determines labor and
Table 1: Values of the Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$ Steady-state households’ utility discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_1,\sigma_2$ Consumption curvature parameter</td>
<td>1.50</td>
</tr>
<tr>
<td>$\nu_1,\nu_2$ Inverse labor supply elasticity parameter</td>
<td>0.50</td>
</tr>
<tr>
<td>$\alpha_1,\alpha_2$ Elasticity of output with respect to labor</td>
<td>2/3</td>
</tr>
<tr>
<td>$\varepsilon_1,\varepsilon_2$ Elasticity of substitution between products</td>
<td>2.54</td>
</tr>
<tr>
<td>$p_R$ Interest rate smoothing</td>
<td>0.90</td>
</tr>
<tr>
<td>$\phi_\pi$ Response of the interest rate to inflation</td>
<td>1.50</td>
</tr>
<tr>
<td>$\rho_R$ Persistence of the Health Shock</td>
<td>0.50</td>
</tr>
<tr>
<td>$\chi$ Disutility of labor (Sector 1)</td>
<td>148.73</td>
</tr>
<tr>
<td>$\gamma$ Disutility of labor (Sector 2)</td>
<td>56.06</td>
</tr>
<tr>
<td>$\phi$ Scaling parameter of utility of consumption (Sector 2)</td>
<td>3.78</td>
</tr>
<tr>
<td>$d_2$ Complementarity between the state of health and consumption (Sector 2)</td>
<td>-1.00</td>
</tr>
<tr>
<td>$m_2$ Complementarity between the state of health and labor (Sector 2)</td>
<td>-(1/3)</td>
</tr>
<tr>
<td>$\eta_1,\eta_2$ Price rigidity</td>
<td>17.92</td>
</tr>
</tbody>
</table>

Note: This table summarizes the values of the parameters in the benchmark analyses. Parameters $\phi_\pi = (1 - \rho_R)\rho_\pi$ and $\phi_\pi = (1 - \rho_R)\rho_\pi$.

output, and 3) with intermediate values of price rigidity, both effects matter for the behavior of the economy. Since, in the latter case, we cannot analytically determine the relative strength of each channel, we now turn to numerical evaluation. We start with describing the parameterization of the model and then turn to first-pass numerical analyses as well as impulse responses.

4.1 Calibration

Table 1 presents a summary of the parameter values. In the benchmark analysis, we let the two sectors be similar in most aspects and set most parameter values to be the same across sectors. As such, any differences between the sectors in their responses to the health shock cannot be attributable to the parameterization of the model.

The time unit is a quarter and the discount factor $\beta$ is set such that the steady-state annual interest rate is roughly 4%. The parameters $\nu_1$ and $\nu_2$ are set such that the labor supply elasticities are 2. The consumption curvature parameters $\sigma_1$ and $\sigma_2$ are set to the middle of the standard values that are assumed in the literature.

The value of $\alpha$ and the parameters of the Taylor rule are standard in the literature, and $\varepsilon_1$ and $\varepsilon_2$ are consistent with a price markup of 10%. Following Faia and Monacelli (2007), we map the price duration to each adjustment cost parameter using the relationship $\eta_i = \theta(\theta-1)(\varepsilon_i^{-1})$ for $i = 1, 2$, with $\theta$ being the quarterly price duration. In short, the price rigidity parameter is pinned down so that the slope of the Phillips curve in a linearized model with Calvo (1983)’s price rigidity is equal to the slope of the Phillips curve in a linearized model with Rotemberg (1982)’s price rigidity. In addition, we assume a price duration of 4 quarters. Since we study impulse responses to a health shock, we leave total factor productivity of each sector at its steady-state level. Therefore $z_{1,t} = z_{2,t}$,
and we normalize the steady-state values to 1. Furthermore, labor utilization is normalized to 1 at the steady state ($\pi_2 = 1$).

We now turn to the parameterization of $\chi, \gamma$ and $\phi$. The starting point is to think about a target for total labor supply ($n_t = n_{1,t} + n_{2,t}$) at the steady state. We let $\pi = 0.21$, which corresponds to a workweek of roughly 35 hours. Second, according to the Employment Situation Report of the Bureau of Labor Statistics, employment in the private sector dropped in March and April 2020 by 21.2 millions, and then rebounded in May 2020. As such, we think of March and April 2020 as capturing the effect of the pandemic on the labor market on impact.

We base the calibration on the Employment Situation Reports of March and April 2020 by identifying the sectors that saw the sharpest declines in employment. As shown in Table B.1, these sectors accounted for 80% of the drop in employment. In addition, in 2019 (the last year before the COVID-19 pandemic), employment in these sectors accounted for nearly 76% of total employment in the private sector and for nearly 63% of the value-added. On this basis, we think of the employment of Sector 2 as accounting for 76% of total labor. Therefore, we approximately obtain $\pi_1 = 0.05$ and $\pi_2 = 0.16$.

Third, to obtain the steady-state value of the relative price ($\bar{p}_2$), we calculate the weighted average of the price indices in 2019 of all sectors that are shown in Table B.1 (which would be a proxy for $P_2$), and then the weighted averages of the remaining sectors (as a proxy for $P_1$). The relative price that we obtain is $\bar{p}_2 = 1.17$. Fourth, we proceed as shown in Appendix A.5. The parameter values and $n_1$ first give $\chi$. We then use the value of $\chi$ to pin down the value of $\gamma$. The steady-state values $\pi_1, \pi_2$ and $\bar{p}_2$ give us the value of $\phi$. These values are reported in Table 1.

To measure the state of public health ($H_t$), we use the excess deaths as a result of COVID-19 that are provided by the Centers for Disease Control and Prevention (CDC). The CDC publishes weekly estimates about the lower bound and the upper bound of excess deaths as well as the average expected death count for each week. The excess rate is then the ratio of the number of excess deaths to the average expected count. As Figure B.1 shows, there has been a spike during March and April 2020, the worst months to date of the pandemic. We use the mean of the excess deaths rates in our analysis.

In March and April 2020, the total excess deaths rate was 35.64%, which would serve as a measure of the decline in $H_t$ in our benchmark analysis. During the same period, the total drop in employment was 16.51% relative to its pre-pandemic level. As such, the change in total labor relative to the change in the state of public health is $dn_t/dH_t = 0.463$. In our calibration, we target this ratio as well as the drop in labor during these two months. In addition, given the lack of sufficient information to calibrate $d_2$ and $m_2$, our analyses assume $d_2(1 - \sigma_2) = m_2(1 + \nu_2)$ so that the coefficients of the state of public health in conditions (22)-(23) are the same for $\phi_t$ and $\gamma_t$. In other words, our starting point is not to make either effect stronger, but rather let the numerical results determine the relative strength of each effect. The benchmark values of $d_2$ and $m_2$ imply that the elasticities of $\phi_t$ and $\gamma_t$ with respect to $H_t$ are -0.5. Also, in the benchmark analysis, we set the standard deviation of the health shock so that the fall in total labor ($n$), with both effects in place, equals the observed fall in labor in March and April 2020; namely, -16.51%.

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Furthermore, we allow for a moderate persistence of the health shock $\rho_H$. With the benchmark value of this parameter, the state of public health returns to its steady-state value after nearly two years of the initial shock. This corresponds to COVID-19 potentially requiring social distancing until 2022, which is in line with some current projections (e.g. Kessler et al. (2020) and Moore et al. (2020)). In what follows, we consider other values of this parameter as well as $d_2$ and $m_2$. As part of the robustness analyses, we also consider a smaller-sized shock to the state of public health.

4.2 Illustrative Numerical Analyses

We start with results that are based on the simplified analyses with fully flexible and fully rigid prices of Section 3. In particular, we estimate the cumulative effect of a health shock on output of the pandemic-sensitive sector ($y_2$). We consider three different values of the persistence parameter ($\rho_H$) and two values for both $d_2$ and $m_2$ (Figure 1). The impact and cumulative effects of the health shock on $y_2$ are increasing functions of the persistence parameters, the sensitivity of labor supply to Sector 2 and the demand for Sector-2 products to the state of public health.\(^5\) The cumulative effect also exponentially increases with the duration of the shock ($T$). Furthermore, for all parameter values that are considered in this exercise, the effects of the health shock in the fully rigid case and the fully flexible case are comparable.

\[ \text{Figure 1: The cumulative effect of a health shock on output of Sector 2.} \]

Note: Top panel: fully flexible prices. Bottom panel: fully rigid prices. The analyses are based on equations (40) and (42).

\(^5\)One may also calculate the corresponding elasticities ($\tilde{\gamma}_2, \tilde{\phi}_2$) by multiplying each value by $\tilde{H}/\tilde{y}_2$, which is 3.39. Furthermore, $m_2 = -1$ and $d_2 = -3$ and gives elasticities of -1.5 for both $\gamma_t$ and $\phi_t$ with respect to $H_t$. 

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4.3 Impulse Responses

We now discuss the impulse response functions that are obtained from solving the full non-linear model with sticky prices. Figure 2 shows the responses of labor (top), output (middle) and inflation (bottom) in each sector as well as at the aggregate level following a decline in $H_t$ that allows for matching the drop in total labor as occurred in March and April 2020. We consider three different cases: the health shock has a labor supply effect only (variable $\gamma_t$ and constant $\phi_t$), a demand effect only (variable $\phi_t$ and constant $\gamma_t$), and both effects. At this stage, we assume that the discount factor does not change following the health shock ($\zeta_t = 1$).

![Graphs of impulse responses](image)

**Figure 2:** Impulse responses to a negative shock to the state of public health ($H_t$).

Note: Percentage deviations from the deterministic steady state. Supply Effect Only- variable $\gamma_t$ and constant $\phi_t$. Demand Effect Only- variable $\phi_t$ and constant $\gamma_t$. Supply and Demand Effects- variable $\gamma_t$ and $\phi_t$. The results of second-order approximation around the steady state.

In all cases, labor and output of Sector 2 fall. As expected, the biggest fall occurs when both effects are operative, in which case labor of Sector 2 falls by more than 20% relative to its steady-state value. Given the concavity of output in labor (as $\alpha < 1$), output of Sector 2 falls by less than one-for-one with labor, but the drop is significant. Interestingly, the declines in labor and output with only a demand effect exceed the corresponding declines when only labor supply is negatively
influenced by the health shock. Nearly 67% of the decline of labor of Sector 2 can be attributed to the demand shock. For total labor, nearly 60% of the fall is due to the demand shock. The shares for output of sector 2 are very similar (66% and 53% for $y_2$ and $y$, respectively).

Labor and output of Sector 1 falls the most when labor supply in Sector 2 falls. The decline in labor of Sector 2, without a direct negative effect on the demand for Sector-2 products, reduces total labor income of the households, which, in turn, reduces demand for both types of products. When only the demand for Sector-2 products falls, one can see two potential opposing effects on the demand for Sector-1 products. First, an income effect due to a (muted) decline in labor of Sector 2. This effect in isolation induces a fall in the demand in Sector 1. Second, a substitution effect as households move away from products of Sector 2 to products of Sector 1, which calls for a rise in the demand for the products of Sector 1. In the benchmark calibration, the second effect dominates and we observe an increase in the demand for the products of Sector 1.

When both supply-side and demand-side effects of a decline in $H_t$ are present, the decline in the labor of Sector 2 is sufficiently large that the income effect dominates the substitution effect, and thus the demand for the products of Sector 1 falls. While labor and output of Sector 2 always fall, the effects on Sector 1 depend on the exact scenario at hand, but this sector mostly sees a decline in activity, too.

In Sector 1, the behavior of inflation follows that of its output. In Sector 2, inflation rises with a negative supply-side effect only, falls with a negative demand-side effect only, and remains largely unchanged with both effects. Since aggregate inflation ($\pi$) is heavily tilted towards Sector 2, $\pi$ behaves mostly like inflation of Sector 2.

Finally, Figure 3 displays the fraction of the changes in labor of Sector 2 and total labor that can be attributed to the demand and supply shock for the first 4 quarters. The share of the demand effect is initially larger. The supply effect then dominates, but the demand effect remains well above zero (particularly for Sector 2). Furthermore, while initially these shares add up to one, the sum for longer horizons drops slightly below one for Sector 2. This might be caused by potential complementarities between the demand and supply effect that disappear over longer horizons when each effect is introduced separately.

5 Robustness Checks

5.1 Smaller Shocks

In this section, we present results with a considerably smaller size of the shock; in particular, we consider a decline of 1% in the state of public health. By doing so, we generalize our analyses as well as overcome any potential drawbacks of using a second order approximation with a very large shock. As is well known, this solution methodology performs considerably better with smaller shocks than larger shocks.

The results, as shown in Figure B.2, are very similar to what we obtain in the benchmark analysis. Most noticeably, the response of the economy with a only demand effect is significantly
bigger than that with a supply-side effect. Furthermore, the demand effect alone generates a drop in total labor that better matches the data. The supply effect alone accounts for nearly 40% of the drop in total labor. As in the benchmark analysis, the aggregate inflation does not always fall following the health shock.

5.2 Introducing a Taste Shifter

While the analyses above well capture the dynamics of labor and output, the behavior of inflation when both effects are considered is an exception. In particular, we do not observe a clear drop in the aggregate inflation rate. In this subsection, we add a taste shifter ($\zeta_t$), i.e. changes in the discount factor, to the benchmark model. Since, at this stage, we do not have sufficient information to estimate the change in the discount factor, we consider various values of $q$ in equation (29). Specifically, we use -0.1 (which is one fifth of the coefficients of $\phi_t$ and $\gamma_t$), -0.25 (one half of the coefficients of $\phi_t$ and $\gamma_t$) and -0.5 (equals the coefficients of $\phi_t$ and $\gamma_t$). On impact, these values of $q$ indicate an increase in the discount factor to 0.9910, 0.9926 and 0.9952, respectively.

Figure B.3 presents the results. In all cases, labor, output and the inflation rates of both sectors decline. Furthermore, Sector 2 is considerably more affected than Sector 1. The bigger drop in activity in Sector 2 and the drop in activity in Sector 1 reflect a desire by households to raise saving and delay consumption of both types of products due to pandemic-induced uncertainty.

5.3 The Role of Price Rigidity

We elaborate on the role of price rigidity in this subsection. Figure B.5 displays the response of labor in Sector 2 and total labor with demand-side effects only and with supply-side effects only under both fully flexible prices and fully sticky prices in Sector 2.
With fully flexible prices of Sector 2, the adjustment in prices prevents a larger drop in labor, and the supply-side effect is larger than the demand-side effect. On the other hand, with fully rigid prices, the drop in labor is larger and the demand-side effect is stronger. In fact, one could attribute all the effects of the health shock to the demand effect with fully sticky prices. These results, together with our benchmark analysis, suggest that, for intermediate values of price rigidity, both effects matter, with the supply-side effect being less important as prices become more rigid. These results are consistent with our analytical analyses in Section 3.

5.4 Alternative Monetary Policy Rule

At the onset of COVID-19, the Federal Reserve System reduced the Federal Funds Rate, and it is expected to remain unchanged through 2022. In this experiment, we let the nominal interest rate be equal to its steady-state value with a probability $\varsigma$ and revert to the flexible interest-rate case, where it follows a Taylor-type rule as in condition (28), with a probability $1 - \varsigma$. The relation between the duration ($T$) of this policy and the probability that the nominal interest rate does not change is given by $T = 1/(1 - \varsigma)$.

Assuming a duration of 8 quarters, we obtain $\varsigma = 0.875$. The impulse responses following a 1% decline in the state of public health is shown in Figure B.4, and they are consistent with our findings in Figure B.2. The only difference is the slightly smaller impact of the health shock on labor and output, which reflects a more accommodating monetary policy. Our main result still holds – the demand-side effect continues to be considerably stronger than the supply-side effect both on impact and for short horizons.

5.5 Constant Elasticity of Substitution

We assume now that the utility function is given by:

$$U(c_t, n_{1,t}, n_{2,t}) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_t^{1+\nu_1} - \gamma_t \left(\frac{w_{2,t} n_{2,t}}{1 + \nu_2}\right)^{1+\nu_2}$$  \hspace{1cm} (44)

and total consumption features the following constant elasticity of substitution (CES):

$$c_t = \left(\phi c_{1,t}^{\omega_1} + (1 - \omega)\phi c_{2,t}^{d_2} c_{2,t}^{\rho 1}\right)^{\frac{1}{\rho}}$$  \hspace{1cm} (45)

where $d_2 > 0$ and $\rho$ is the elasticity of substitution. With $\rho \rightarrow 0$, $c_{1,t}$ and $c_{2,t}$ are perfect complements. For $\rho \rightarrow 1$, we obtain a Cobb-Douglas aggregator, and with $\rho \rightarrow \infty$, the two products are perfect substitutes. In addition, we set $\varepsilon_i > \rho$ for $i = 1, 2$ so that the within-sector substitutability is greater than between-sector substitutability. The first-order conditions then give:

$$\frac{c_{2,t}}{c_{1,t}} = \frac{(1 - \omega)\phi H_t^{d_2(\rho-1)}}{\omega p_{2,t}^{jd}}$$  \hspace{1cm} (46)

which is equivalent to condition (24). Also, in this case:
\[ \phi_t = \phi H_t^{d_2(\frac{\sigma_1}{\rho})} \]  

(47)

For the substitution between \( c_{1,t} \) and \( c_{2,t} \) (\( \rho > 1 \)), a decline in the state of public health reduces the desire to consume \( c_{2,t} \) and shifts consumption towards Sector 1.

Before turning to the numerical results, we make two comments. First, with this CES specification, one could use logarithmic preferences while still keeping the demand-side effect in place. Therefore, qualitatively, our results do not hang on the assumption of \( \sigma_2 > 1 \). Second, in Appendix A.6 we demonstrate that our analytical results can be generalized. Specifically, by setting \( \rho \sigma_i = 1 \), we obtain the same results as in Section 3.

Numerically, we first show the results with \( \rho = 2/3 \) so that the elasticity of substitution in the model with a CES aggregator equals the elasticity of substitution in the benchmark model \((1/\sigma_i)\). The results, displayed in Figure B.6, confirm our benchmark results.

In the benchmark analysis, the drop in the demand for \( c_{1,t} \) following a decline in labor supply to Sector 2 could be attributed to both a decline in labor income of the household and the complementarity between sectors. In this section, we illustrate that this result holds even when the products of Sector 1 and Sector 2 are substitutes. Figure B.7 displays the behavior of labor, output and inflation of Sector 1 with the supply-side effect only \((d_2 = 0)\). We consider three values for the elasticity of substitution: \( \rho = 2/3 \) as assumed above, \( \rho = 1.1 \) (an elasticity of substitution that is slightly above 1) and a relatively high elasticity of substitution \((\rho = 2)\).

In all cases considered, activity in Sector 1 falls, particularly when the products of the two sectors are complements. The key finding, however, is that a decline in the activity of Sector 1 could occur when the products are substitutes. This result reflects a strong negative income effect: the fall in labor supply of Sector 2 reduces total labor and total output (income). Households respond by reducing their demand for the products of both sectors. Essentially, there are two effects operating on the consumption of Sector 1. The income effect in itself reduces the demand for Sector-1 products while the substitution effect raises the demand for these products. For lower values of \( \rho \), the income effect dominates. As \( \rho \) increases, the income effect becomes weaker, and as a result, the negative effect of the shock on consumption of Sector 1 is reduced.

### 5.6 Endogenous State of Public Health \((H_t)\)

In this subsection, we let the state of public health be a negative function of the activity in Sector 2, as follows:

\[
\ln \left( \frac{H_t}{\hat{H}} \right) = \rho_H \ln \left( \frac{H_{t-1}}{\hat{H}} \right) + (1 - \rho_H) \varphi \left( \frac{y_{2,t}}{\hat{y}_2} \right) + \nu_{H,t} \tag{48}
\]

where \( \varphi < 0 \). The results are shown in Figure B.8, and they are very similar to our benchmark results. Therefore, our main findings regarding the strength of each channel are robust to the choice of the state of public health process.
5.7 Reallocation in a Three-Sector Model

Barrero et al. (2020) document that for each 10 jobs that have been lost due to COVID-19, 3 jobs were created. In this section, we consider a three-sector economy that illustrates their findings. Sector 1 is as in the benchmark analysis; it continues to be the least vulnerable to pandemics. Sector 2 is the most vulnerable, and Sector 3 benefits from the lockdown and social distancing that are induced by a pandemic. Examples of Sector 3 are online movie streaming, grocery delivery and food delivery.

In this setup, we have three levels of substitution: between products in the same sector (with the elasticities being $\varepsilon_1, \varepsilon_2$), between Sector 2 and Sector 3 ($\tau$), and between Sector 1 and the other two sectors ($\rho$). The highest substitution is within sector, and the lowest is between Sector 1 and the other two sectors. Therefore, $\varepsilon_i > \tau > \rho$.

Total consumption ($c_t$) is now given by the following CES function:

$$c_t = \left( \omega c_{1,t} + (1-\omega) \left[ \delta \phi_2 \left( H_t^{d_2 c_{2,t}} \right)^{\frac{\tau-1}{\tau}} + (1-\delta) \phi_3 \left( H_t^{d_3 c_{3,t}} \right)^{\frac{\tau-1}{\tau}} \right] \right)^{\frac{\rho}{\tau}}$$

with $\tau$ being the elasticity of substitution between $c_{2,t}$ and $c_{3,t}$, and $d_2 > 0, d_3 < 0$, and other variables and parameters as above. From the first-order conditions we then obtain:

$$\frac{c_{3,t}}{c_{2,t}} = \left( \frac{1-\delta}{\delta} \phi_3 \left( \frac{H_t^{d_3}}{H_t^{d_2}} \right)^{(\tau-1)} \left( \frac{p_{2,t}}{p_{3,t}} \right)^{\tau} \right)$$

and we can also express the utility preference variables as:

$$\phi_{2,t} = \phi_2 H_t^{d_2(\frac{\tau-1}{\tau})}$$

$$\phi_{3,t} = \phi_3 H_t^{d_3(\frac{\tau-1}{\tau})}$$

In relative terms, the demand for the products of Sector 3 will rise following a negative shock to public health. This could occur because of a rise in the demand for Sector-3 products and/or a fall in the demand for Sector-2 products. In either case, the health shock could lead to an allocation of resources between these two sectors. We illustrate this channel with $d_3 = 0$ so that the shift to Sector 3 occurs as a result of a negative shock in Sector 2.

In April 2020, the Employment Situation Report shows an increase in labor of the following industry: Couriers and messengers (part of the Transportation and Warehousing super sector), General merchandise stores, including warehouse clubs and supercenters (part of the Retail Trade super sector), and Other information services (part of the Information super sector). In 2019, the share of these sectors in total private-sector employment was roughly 2.4%. As such, we let $(1-\omega)(1-\delta) = 0.024$, implying $\delta = 0.968$ for our numerical analysis.
The numerical results are presented in Figure B.9. Labor and output of Sector 1 and Sector 2 fall. On the other hand, both output and labor of Sector 3 rise in response to the health shock. Since Sector 3 is relatively small, total labor and total output both fall, but by less than that in the benchmark model. In summary, the positive impact that the shock has on Sector 3 mitigates some of the health shock on the labor market and aggregate output.

6 Conclusions

The outbreak of COVID-19 has given rise to heated discussions about the nature of this health shock and its impacts on the economy. We propose multiple channels through which a health shock can affect economic activity in a two-sector model, where one sector is particularly sensitive to pandemics. Following a negative health shock, the supply of labor to the pandemics-sensitive sector falls, the demand for the products of this sector falls, and the uncertainty rises.

Both demand-side and supply-side effects matter. We demonstrate that the demand-side effects are considerably stronger for shorter horizons (within 2-4 quarters) while supply-side effects are stronger for longer horizons. The demand effects, however, remain meaningful for longer horizons. Price rigidity plays a key role: the more rigid prices are, the more important the demand channel is. Furthermore, we extend the benchmark model to allow for a reallocation channel through which a health shock shifts resources from the most pandemics-sensitive sector to a third sector that actually benefits from the pandemics.

Our findings have potential implications for policy making. Fiscal stimulus, an accommodative monetary policy and boosting confidence could be useful to counter the demand-side effect of the shock, at least in the short run. Commitment to global supply chains and providing businesses with credit could mitigate the supply-side effect of the pandemic.

References


A Mathematical Appendix

A.1 The Production Sector

We show here the derivations of the labor demand and Phillips curve conditions. For illustration, we show the steps for Sector 1, but similar steps yields the optimality conditions for firms in Sector 2. As standard in the literature, one can proceed in two steps. First, each firm $j$ in Sector 1 chooses labor subject to the demand condition for its products ($y_{1,j,t}^d$). The firm then solves the following problem:

$$
\min w_{1,j,t} n_{1,j,t}
$$

subject to:

$$
z_{1,j,t} n_{1,j,t}^\alpha \geq y_{1,j,t}^d
$$

Denoting the Lagrange multiplier on condition (A.2) by $mc_{j,1,t}$ and taking first-order condition with respect to $n_{1,j,t}$, the solution gives the following labor demand function:

$$
\alpha mc_{1,j,t} z_{1,t} n_{1,j,t}^{\alpha - 1} = w_{1,j,t}.
$$

which could also be written as $\alpha mc_{1,j,t} y_{1,j,t} = w_{1,j,t} n_{1,j,t}$. The multiplier $mc_{1,j,t}$ measures the contribution of one additional unit of output to the revenue of firm $j$ and, in equilibrium, it equals the real marginal cost of the firm. Imposing symmetry across firms then gives condition (12).

Second, the first chooses its price $P_{1,j,t}$ so as to:

$$
\max_{P_{1,j,t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\xi_t \lambda_t}{\zeta_t \lambda_0} \left( \frac{P_{1,j,t}}{P_{1,t}} - \alpha mc_{1,j,t} y_{1,j,t} - \frac{\eta_1}{2} \left( \frac{P_{1,j,t}}{P_{1,j,t-1}} - 1 \right)^2 y_{1,t} \right) \right\}
$$

subject to:

$$
y_{1,j,t} \geq \left( \frac{P_{1,j,t}}{P_{1,t}} \right)^{\varepsilon_i} y_{1,t}^d
$$

The last term in (A.4) captures the cost of adjusting prices as in Rotemberg (1982), where the costs are expressed in units of the aggregate final good ($y_{1,t}$), and $\eta_1$ is the parameter that governs the degree of price rigidity. In a symmetric equilibrium, in which all firms set the same price in equilibrium, the optimal choice of prices leads to the following standard forward-looking Phillips curve:

$$
1 - \eta_1 (\pi_{1,t} - 1) \pi_{1,t} + \beta \eta_1 \mathbb{E}_t \left[ \frac{\lambda_{t+1} \xi_{t+1}}{\lambda_t \xi_t} (\pi_{1,t+1} - 1) \pi_{1,t+1} \frac{y_{1,t+1}}{y_{1,t}} \right] = \varepsilon_i (1 - \alpha mc_{1,t}).
$$

which is condition (13) in the text.
A.2 Obtaining the Expressions for $\gamma_t$ and $\phi_t$

Start with the following form of the utility function:

$$U = \frac{c_{1,t}^{1-\sigma_1}}{1-\sigma_1} + \phi H_t^{d_2(1-\sigma_2)} \frac{c_{2,t}^{1-\sigma_2}}{1-\sigma_2} - \chi t n_{1,t}^{1+\nu_1} - \gamma H_t^{m_2(1+\nu_2)} \frac{(u_{2,t}n_{2,t})^{1+\nu_2}}{1+\nu_2}$$  \hspace{1cm} (A.7)

which suggests that the consumption of $c_{2,t}$ and the supply of labor to Sector 2 ($n_{2,t}$) depend on the state of public health. Put differently, there is some complementarity between public health and consumption and labor in this sector.

Re-write as:

$$U = \frac{c_{1,t}^{1-\sigma_1}}{1-\sigma_1} + \phi H_t^{d_2(1-\sigma_2)} \frac{c_{2,t}^{1-\sigma_2}}{1-\sigma_2} - \chi t n_{1,t}^{1+\nu_1} - \gamma H_t^{m_2(1+\nu_2)} \frac{(u_{2,t}n_{2,t})^{1+\nu_2}}{1+\nu_2}$$  \hspace{1cm} (A.8)

Next, define $\phi_t = \phi H_t^{d_2(1-\sigma_2)}$ and $\gamma_t = \gamma H_t^{m_2(1+\nu_2)}$ to get:

$$U = \frac{c_{1,t}^{1-\sigma_1}}{1-\sigma_1} + \phi_t \frac{c_{2,t}^{1-\sigma_2}}{1-\sigma_2} - \chi_t n_{1,t}^{1+\nu_1} - \gamma_t \frac{(u_{2,t}n_{2,t})^{1+\nu_2}}{1+\nu_2}$$  \hspace{1cm} (A.9)

which is the utility function that is presented in (1).

In order for a decline in the state of public health to reduce the desire to consume $c_{2,t}$ and because $\sigma_2 > 1$, we need $d_2 < 0$. And, for the labor supply in Sector 2 to fall following a decline in the state of public health, we need $m_2 < 0$.

The alternative way to introduce the state of public health is as follows:

$$U = u(c_{1,t}) + \phi_t u(c_{2,t}) - \chi_t v(n_{1,t}) - \gamma_t v(n_{2,t})$$  \hspace{1cm} (A.10)

and then relate $\phi_t, \gamma_t$ to the state of public health. In this case, one may use logarithmic preferences and obtain similar qualitative results to what we are presenting in the text.

A.3 Effects of the Health Shock on Productivity

Suppose now that productivity in Sector 2 depends on the state of public health: $z_{2,t} = H_t^{a_2}$, with $a_2 > 0$. In log deviations: $\varepsilon_{2,t} = a_2 \Pi_t$. The labor supply in Sector 2 is not directly affected by the pandemic; therefore, $\gamma_t = 1$ for all $t$.

In log deviations, households optimization gives:

$$\nu_2 \bar{n}_{2,t} = -\sigma_1 \bar{c}_{1,t} + \bar{w}_{2,t}$$ \hspace{1cm} (A.11)

And the labor demand is now given by:

$$\bar{m}\bar{c}_{2,t} + \bar{z}_{2,t} + (\alpha_2 - 1)\bar{n}_{2,t} = \bar{w}_{2,t}$$ \hspace{1cm} (A.12)
Consider now fully flexible prices. As in the text, there is no effect on Sector 1. Therefore $\tilde{c}_{1,t} = 0$. Then, combining conditions (A.11)-(A.12) gives:

$$(1 + \nu_2 - \alpha_2)\tilde{z}_{2,t} = \tilde{z}_{2,t}$$

(A.13)

or, using the production function and the relation between $z_{2,t}$ and $H_t$:

$$\frac{\tilde{y}_{2,t}}{\tilde{H}_{2,t}} = \frac{a_2 \alpha_2}{1 + \nu_2 - \alpha_2} > 0$$

(A.14)

which is the equivalent to (32). Labor and output in Sector 2 are determined only by the supply-side effect of the health shock.

This modification does not change the Euler conditions; therefore, they continue to be given by (34)-(35). As such, with fully rigid prices, labor and output in Sector 2 are determined only by the demand-side effect of the health shock.

A.4 Calibrating the Steady-State Values of $\gamma_t$, $\phi_t$ and $\chi_t$: General

In this appendix, we discuss the parameterization of $\gamma, \phi$ and $\chi$ at the steady state. The steady-state versions of the households’ optimality conditions are given by (with $\overline{w}_2 = 1$):

$$\overline{c}_{1,1}^{\sigma_1} = \bar{\lambda}$$

(A.15)

$$\phi \overline{c}_{2,2}^{\sigma_2} = \bar{\lambda} \overline{p}_2$$

(A.16)

$$\chi \overline{n}_{1} = \bar{\lambda} \overline{w}_1$$

(A.17)

$$\gamma \overline{n}_{2} = \bar{\lambda} \overline{w}_2$$

(A.18)

and the labor demand conditions:

$$\alpha_1 \overline{m} \overline{c}_1 \overline{z}_1 \overline{n}_{1} = \overline{w}_1$$

(A.19)

$$\alpha_2 \overline{m} \overline{c}_2 \overline{z}_2 \overline{n}_{2} = \overline{w}_2$$

(A.20)

At the steady state $\overline{w}_1 = \overline{w}_2 = 1$. Then, the real marginal costs are given by $\overline{m} \overline{c}_1 = \frac{\varepsilon_{1,-1}}{\alpha_1 \varepsilon_1}$ and $\overline{m} \overline{c}_2 = \frac{\varepsilon_{2,-1}}{\alpha_2 \varepsilon_2}$. The real wages are then given by:

$$\overline{w}_1 = \left(\varepsilon_{1,-1} \overline{z}_1 \overline{n}_{1}^{\alpha_1-1} \right)^{\frac{1}{\varepsilon_1}}$$

(A.21)

$$\overline{w}_2 = \left(\varepsilon_{2,-1} \overline{z}_2 \overline{n}_{2}^{\alpha_2-1} \right)^{\frac{1}{\varepsilon_2}}$$

(A.22)

Combine (A.15) and (A.17):

$$\chi \overline{n}_{1} = \overline{w}_1$$

(A.23)
or using the production function and market clearing in Sector 1, \( \bar{z}_1 = \bar{z}_2 \bar{n}_1^{\alpha_1} \):

\[
\chi \bar{n}_1^{\nu_1} (\bar{z}_1 \bar{n}_1^{\alpha_1})^{\sigma_1} = \bar{w}_1
\]  \hspace{1cm} (A.24)

Combine (A.21) and (A.24) to obtain:

\[
\chi = \left( \frac{\varepsilon_1 - 1}{\varepsilon_1} \right) \bar{z}_1^{1-\sigma_1} \bar{n}_1^{[\alpha_1(1-\sigma_1)-(1+\nu_1)]}
\]  \hspace{1cm} (A.25)

For \( \gamma \), combine (A.17)-(A.18) and (A.21)- (A.22):

\[
\gamma = \chi \left( \frac{\varepsilon_2 - 1}{\varepsilon_1} \right) \frac{\bar{z}_2 \bar{n}_2^{(\alpha_2-\nu_2-1)}}{\bar{z}_1 \bar{n}_1^{(\alpha_1-\nu_1-1)}}
\]  \hspace{1cm} (A.26)

And, for \( \phi \), combine (A.15) and (A.16) to obtain:

\[
\phi = \bar{p}_2 \frac{\bar{c}_2^{\sigma_2}}{\bar{c}_1}
\]  \hspace{1cm} (A.27)

or using the production function and market clearing in Sector 1:

\[
\phi = \bar{p}_2 \bar{n}_2^{\alpha_2 \sigma_2} \bar{n}_1^{\alpha_1 \sigma_1}
\]  \hspace{1cm} (A.28)

A.5 Calibrating the Steady-State Values of \( \gamma_t, \phi_t \) and \( \chi_t \): Benchmark

In the benchmark analysis, we assume \( \varepsilon_1 = \varepsilon_2, \nu_1 = \nu_2, \sigma_1 = \sigma_2, \alpha_1 = \alpha_2, \bar{z}_1 = \bar{z}_2 = 1 \), and thus drop the sector index. Then:

\[
\chi = \left( \frac{\varepsilon - 1}{\varepsilon} \right) \bar{n}_1^{[\alpha(1-\sigma)-(1+\nu)]}
\]  \hspace{1cm} (A.29)

\[
\gamma = \chi \left( \frac{\bar{n}_2}{\bar{n}_1} \right)^{(\alpha-\nu-1)}
\]  \hspace{1cm} (A.30)

\[
\phi = \bar{p}_2 \left( \frac{\bar{n}_2}{\bar{n}_1} \right)^{\alpha \sigma}
\]  \hspace{1cm} (A.31)
A.6 Analytical Analysis with CES Preferences

Recall that the utility function is given by:

\[ U(c_t, n_{1,t}, n_{2,t}) = \frac{c_t^{1-\sigma}}{1-\sigma} - \chi_t^{\frac{1+\nu_1}{1+\nu_1}} - \gamma_t \frac{(u_{2,t} n_{2,t})^{1+\nu_2}}{1+\nu_2} \]  

(A.32)

and total consumption features the following constant elasticity of substitution (CES):

\[ c_t = \left( \omega c_{1,t}^{\frac{\nu_1}{\nu_1}} + (1-\omega) \phi(R_t^{d_2} c_{2,t}^{\frac{\nu_2}{\nu_2}}) \right)^{\frac{1}{\nu_1}} \]  

(A.33)

The first order conditions read:

\[ \omega c_t \left( \frac{1-\rho_0}{\rho} \right) c_{1,t}^{\frac{1}{\rho}} = \lambda_t \]  

(A.34)

\[ (1-\omega) \phi_t c_t \left( \frac{1-\rho_0}{\rho} \right) c_{2,t}^{\frac{1}{\rho}} = \lambda_t p_{2,t} \]  

(A.35)

\[ \chi_t n_{1,t}^{\nu_1} = \lambda_t w_{1,t} \]  

(A.36)

\[ \gamma_t (u_{2,t} n_{2,t})^{\nu_2} = \lambda_t w_{2,t} \]  

(A.37)

\[ \lambda_t \zeta_t = \beta R_t \mathbb{E}_t \left( \frac{\zeta_{t+1} \lambda_{t+1}}{\pi_{1,t+1}} \right) \]  

(A.38)

We then obtain:

\[ c_t \left( \frac{1-\rho_0}{\rho} \right) c_{1,t}^{\frac{1}{\rho}} \zeta_t = \beta R_t \mathbb{E}_t \left( \frac{\zeta_{t+1} c_{1,t+1}^{\frac{1}{\rho}}}{\pi_{1,t+1}} \right) \]  

(A.39)

\[ c_t \left( \frac{1-\rho_0}{\rho} \right) c_{2,t}^{\frac{1}{\rho}} \zeta_t = \beta R_t \mathbb{E}_t \left( \frac{\zeta_{t+1} p_{2,t} c_{1,t+1}^{\frac{1}{\rho}}}{p_{2,t+1} \pi_{1,t+1}} \right) \]  

(A.40)

\[ \chi_t n_{1,t}^{\nu_1} = \omega c_t \left( \frac{1-\rho_0}{\rho} \right) c_{1,t}^{\frac{1}{\rho}} w_{1,t} \]  

(A.41)

\[ \gamma_t (u_{2,t} n_{2,t})^{\nu_2} = \omega c_t \left( \frac{1-\rho_0}{\rho} \right) c_{1,t}^{\frac{1}{\rho}} w_{2,t} \]  

(A.42)

Let \( u_t = 1 \) and \( \zeta_t = 1 \). Then, Log-linearization and using \( \mathbb{E}_t \bar{x}_{t+1} = \rho_t \bar{x}_t \) and \( \bar{R}_t = \phi_t \bar{x}_{1,t} \) give:

\[ (\phi_\pi - \rho_H) \bar{x}_{1,t} = (\rho_H - 1) \left( \frac{1}{\rho} \bar{c}_{1,t} - \left( \frac{1-\rho \sigma_1}{\rho-1} \right) \bar{c}_t \right) \]  

(A.43)

\[ (\phi_\pi - \rho_H) \bar{x}_{1,t} = (\rho_H - 1) \left( \frac{1}{\rho} \bar{c}_{2,t} + \bar{p}_{2,t} - \bar{\phi}_t - \left( \frac{1-\rho \sigma_2}{\rho-1} \right) \bar{c}_t \right) \]  

(A.44)

\[ \nu_1 \bar{x}_{1,t} = \left( \frac{1-\rho \sigma_1}{\rho-1} \right) \bar{c}_t - \left( \frac{1}{\rho} \bar{c}_{1,t} + \bar{w}_{1,t} \right) \]  

(A.45)
\[ \gamma_t + \nu_2 \bar{n}_{2,t} = \left( \frac{1 - \rho \sigma_1}{\rho - 1} \right) \bar{c}_t - \frac{1}{\rho} \bar{c}_{1,t} + \bar{w}_{2,t} \]  
(A.46)

\[ (\phi \pi - \rho_H) \bar{\pi}_{1,t} = (\rho_H - 1) \left( \frac{1}{\rho} \bar{c}_{2,t} + \bar{p}_{2,t} - \phi_t - \left( \frac{1 - \rho \sigma_1}{\rho - 1} \right) \bar{c}_t \right) \]  
(A.47)

**Fully Flexible Prices:** With fully flexible prices, the real marginal cost is constant. Therefore, \( \bar{w}_{1,t} = (\alpha_1 - 1)\bar{n}_{1,t} \). Using the production function and market clearing in Sector 1 then give:

\[ \nu_1 \bar{n}_{1,t} = \left( \frac{1 - \rho \sigma_1}{\rho - 1} \right) \bar{c}_t - \frac{\alpha_1}{\rho} \bar{n}_{1,t} + (\alpha_1 - 1)\bar{n}_{1,t} \]  
(A.48)

Rearranging gives:

\[ \left( \rho \nu_1 + \alpha_1 + \rho(1 - \alpha_1) \right) \bar{n}_{1,t} = \left( \frac{1 - \rho \sigma_1}{\rho - 1} \right) \bar{c}_t \]  
(A.49)

If \( \rho \sigma_1 = 1 \), then \( \bar{n}_{1,t} = 0 \). Sector 1 does not respond to the health shock with fully flexible prices. This result generalizes our result in Section 3.

For Sector 2, with \( \rho \sigma_2 = 1 \), we obtain:

\[ (1 + \nu_2 - \alpha_2) \bar{n}_{2,t} = -\gamma_t \]  
(A.50)

or, using the production function:

\[ \left( \frac{1 + \nu_2 - \alpha_2}{\alpha_2} \right) \bar{y}_{2,t} = -\gamma_t \]  
(A.51)

which is condition (31) in the text.

**Fully Rigid Prices:** If prices are fully rigid in both sectors, then \( \bar{\pi}_{1,t} = \bar{p}_{2,t} = \bar{\phi}_t = 0 \). Under the assumption \( \rho \sigma_1 = 1 \), condition (A.43) gives \( \bar{c}_{1,t} = 0 \). Sector 1 does not respond to the shock.

For Sector 2, after using the market clearing condition:

\[ \bar{y}_{2,t} = \rho \bar{\phi}_t \]  
(A.52)

which is the equivalent to condition (37) in the text. Therefore, consumption, labor and output of Sector 2 are fully determined by the demand effect of the health shock, thus re-affirming our benchmark results.
## B Tables and Figures

### Table B.1: Sector Statistics

<table>
<thead>
<tr>
<th>Sector</th>
<th>Share in Private-Sector Employment (%)</th>
<th>Share in Value Added (%)</th>
<th>Share in the drop in Employment of March &amp; April 2020 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Retail Trade</td>
<td>12.19</td>
<td>6.75</td>
<td>11.25</td>
</tr>
<tr>
<td>Transportation and warehousing</td>
<td>4.38</td>
<td>3.35</td>
<td>2.69</td>
</tr>
<tr>
<td>Financial activities</td>
<td>6.81</td>
<td>21.53</td>
<td>1.32</td>
</tr>
<tr>
<td>Professional and business services</td>
<td>16.61</td>
<td>15.10</td>
<td>10.83</td>
</tr>
<tr>
<td>Education and health services</td>
<td>18.84</td>
<td>9.93</td>
<td>13.12</td>
</tr>
<tr>
<td>Leisure and hospitality</td>
<td>12.92</td>
<td>4.30</td>
<td>39.25</td>
</tr>
<tr>
<td>Other services</td>
<td>4.59</td>
<td>2.20</td>
<td>1.15</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>76.33</strong></td>
<td><strong>63.17</strong></td>
<td><strong>79.62</strong></td>
</tr>
</tbody>
</table>

*Note: This table summarizes data about the sector that we use as a proxy for Sector 2 in the model. Data source- “The Employment Situation Report”, provided by the Bureau of Labor Statistics.*

![Figure B.1: Lower and higher estimates of excess deaths: weekly data for 2017-2020.](image)

*Note: Data source- “Excess_Deaths_Associated_with_COVID-19”, provided by the Centers for Diseases Control and Prevention (CDC).*
Figure B.2: Impulse responses to a negative one percent shock to the state of public health ($H_t$).

Note: Percentage deviations from the deterministic steady state. Supply Effect Only- variable $n_t$ and constant $\phi_t$. Demand Effect Only- variable $\phi_t$ and constant $n_t$. Supply and Demand Effects- variable $n_t$ and $\phi_t$. The results of second-order approximation around the steady state.
Figure B.3: Impulse responses to a negative one percent shock to the state of public with a variable discount factor.

Note: The values of $q$ at -0.10, -0.25 and -0.50 correspond to a discount factor of 0.9910, 0.9926 and 0.9952, respectively.
Figure B.4: Impulse responses to a negative one percent shock to the state of public health ($H_t$).

Note: Percentage deviations from the deterministic steady state. The nominal interest rate equals its steady-state value with probability 0.875 and reverts to a Taylor-type rule with probability 0.125. Supply Effect Only- variable $\gamma_t$ and constant $\phi_t$. Demand Effect Only- variable $\phi_t$ and constant $\gamma_t$. Supply and Demand Effects- variable $\gamma_t$ and $\phi_t$. The results of second-order approximation around the steady state.
Figure B.5: Impulse responses to a negative one percent shock to the state of public health ($H_t$).

Note: percentage deviations from the deterministic steady state. Top panel- with constant discount factor. Bottom panel- variable discount factor. Supply, Flexible- variable $\gamma_t$ and constant $\phi_t$ with fully flexible prices in Sector 2. Demand, Flexible- variable $\phi_t$ and constant $\gamma_t$ with fully flexible prices in Sector 2. Supply, Rigid- variable $\gamma_t$ and constant $\phi_t$ with fully rigid prices in Sector 2. Demand, Flexible- variable $\phi_t$ and constant $\gamma_t$ with fully rigid prices in Sector 2.
Figure B.6: Impulse responses to a negative one percent shock to the state of public health ($H_t$) for the model with a CES utility function.

Note: percentage deviations from the deterministic steady state. 
Supply Effect Only - variable $\gamma_t$ and constant $\phi_t$. 
Demand Effect Only - variable $\phi_t$ and constant $\gamma_t$. 
Supply and Demand Effects - variable $\gamma_t$ and $\phi_t$. 
All Effects - variable $\gamma_t$, $\phi_t$ and $\zeta_t$, with $q = -0.25$, $(1 - \omega) = 0.63$ and $\rho = 2/3$. 

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Figure B.7: Impulse responses to a negative one percent shock to the state of public health ($H_t$) for the model with a CES utility function and a supply-side effect only (variable $\gamma_t$, constant $\phi_t$ and constant discount factor).

Note: For each case, $d_2$ is adjusted so that $d_2(\rho - 1)/\rho = -0.50$. 
Figure B.8: Impulse responses to a negative shock to the state of public health \((H_t)\).

Note: Percentage deviations from the deterministic steady state. Supply Effect Only- variable \(\gamma_t\) and constant \(\phi_t\). Demand Effect Only- variable \(\phi_t\) and constant \(\gamma_t\). Supply and Demand Effects- variable \(\gamma_t\) and \(\phi_t\). The results of second-order approximation around the steady state. The model with endogenous state of public health \((H_t)\). We let \(\varpi = -1.0\).
Figure B.9: Impulse responses to a negative one percent shock to the state of public health ($H_t$) in the three-sector model.

Note: percentage deviations from the deterministic steady state. $(1 - \omega) = 0.763$ and $\delta = 0.968$. We set $\tau = 2$ so that the elasticity of $\phi_{2,t}$ with respect to $H_t$ is -0.5, as in the benchmark analysis. Preferences are given by equation (44).