A new measure of earnings forecast uncertainty

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ABSTRACT

Relying on the well-established theoretical result that uncertainty has a common and an idiosyncratic component, we propose a new measure of earnings forecast uncertainty as the sum of dispersion among analysts and the variance of mean forecast errors estimated by a GARCH model. The new measure is based on both common and private information available to analysts at the time they make their forecasts. Hence, it alleviates some of the limitations of other commonly used proxies for forecast uncertainty in the literature. Using analysts' earnings forecasts, we find direct evidence of the new measure's superior performance.

1. Introduction

Analysts' forecasts are widely used in the accounting and finance literature to study market participants' expectations. Researchers and investors are especially interested in estimating uncertainty about future earnings, because it reveals important characteristics of the firm's information environment prior to the release of accounting results. Since uncertainty is inherently unobservable, evaluating its estimates poses challenging methodological problems. As a result, researchers have experimented with alternative proxies for earnings forecast uncertainty.

One of the most commonly used measures of earnings forecast uncertainty is the dispersion among analysts (see, e.g. Baginski et al., 1993; Diether et al., 2002; Clement et al., 2003; Yeung, 2009). Dispersion, as a proxy for uncertainty, has several advantages. It is easy to calculate and gives a measure of uncertainty around the time the forecast is made, i.e. in real time. However, Abarbanell et al. (1995) and Johnson (2004), among others, point out that dispersion does not capture uncertainty fully. Indeed, dispersion represents only one element of uncertainty, namely uncertainty arising from analysts' private information and diversity of forecasting models. In addition, Barron et al. (2009) show that the change in dispersion does not indicate a change in uncertainty but rather a change in information asymmetry. As a result, dispersion
tends to be a noisy and unreliable proxy for earnings forecast uncertainty when the uncertainty shared by all analysts becomes dominant or when the change in uncertainty is the construct of interest.

Another popular proxy for uncertainty is the measure proposed by Barron et al. (1998). The authors suggest that uncertainty be estimated as the sum of dispersion and the squared error in the mean forecast (BKLS Uncertainty hereafter). BKLS Uncertainty correctly recognizes the fact that uncertainty is comprised of two components. However, there are important caveats when using BKLS Uncertainty as a proxy for ex ante uncertainty. Since forecast errors are known to respondents only after the announcement of actual earnings, BKLS Uncertainty provides an estimate of ex post uncertainty. As discussed in detail later in this paper, BKLS Uncertainty is excessively affected by significant unanticipated events following the forecast, such as 9/11-type disasters, bankruptcy and restructuring. In addition, it relies on the assumption that actual earnings are exogenous, which is unlikely to hold in practice, as there is an extensive stream of research showing that managers manipulate earnings to meet or beat analysts’ forecasts (see, e.g. DeGeorge et al., 1999; Abarbanell and Lehavy, 2003). To the extent that the exogeneity assumption of actual earnings is violated, the squared error in the mean forecast will be understated and the resulting BKLS Uncertainty will also be understated. Hence, the reliability of BKLS Uncertainty as a proxy for ex ante uncertainty faced by analysts becomes an important empirical issue.

In this paper we propose a new empirical measure of earnings forecast uncertainty. Since uncertainty is fundamentally an ex ante concept attached to a forecast before actual earnings are known, it must be constructed using data available in real time. Accordingly, we estimate the variance of mean forecast errors using a Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model, conditional on the information known to financial analysts when making their earnings forecasts. Specifically, this method provides an estimate of the volatility in analysts’ common forecast errors in the current period based on historical mean forecast errors. The new measure is then constructed as the sum of the projected variance of mean forecast errors, estimated by a GARCH model, and dispersion among financial analysts. Compared to other existing measures in the literature, the new uncertainty measure has the advantages that it is estimated using information known to analysts at the time they make forecasts and it captures both components of the theoretical construct.

We examine the performance of the new measure empirically using the I/B/E/S data set during 1984–2007. Our analysis shows that dispersion alone tends to understate uncertainty, especially at longer horizons. BKLS Uncertainty, on the other hand, tends to become unduly volatile in periods of unanticipated significant events and understates uncertainty in other periods. In contrast, the new measure gives an appropriate measure of uncertainty faced by financial analysts under these circumstances. For a further comparison, we test the hypothesis that analysts rely more on reported earnings to revise their forecasts when the ex ante uncertainty in future earnings is relatively high. Since our uncertainty measure is expected to be superior to the other two especially at longer horizons, we predict that the positive relation between forecast revisions and earnings surprises is stronger when uncertainty is measured by the new proxy. Our empirical results are consistent with this prediction and provide further evidence of the appropriateness and superiority of the new empirical proxy.

The rest of the paper is organized as follows. Section 2 discusses two commonly used proxies for uncertainty and proposes a new measure. Section 3 describes the empirical experiment and presents the estimation results. Section 4 concludes. Additional details on the GARCH model estimation are provided in Appendix A.

2. Empirical measures of earnings forecast uncertainty

Before discussing the possible empirical proxies for uncertainty, one must understand the theoretical construct. We adopt the well-known model of Barron et al. (1998). Let \( H_{ith} \) be the \( h \)-quarter ahead earnings forecast made by analyst \( i \), for target year \( t \), where \( i = 1, \ldots, N \), \( t = 1, \ldots, T \) and \( h = 1, \ldots, H \), and \( \mu_t \) be the mean forecast averaged over \( N \) analysts. Then the observed dispersion among analysts, \( d_{ith} \), is expressed as the population variance of their point forecasts:

\[
d_{ith} = \frac{1}{N} \sum_{i=1}^{N} (\mu_{ith} - \mu_{th})^2.
\]

(1)

Let \( y_t \) denote actual earnings. The observed error in the mean forecast, \( e_{ith} \), is defined as

\[
e_{ith} = y_t - \mu_{ith}.
\]

(2)

Barron et al. (1998) find that after observing forecasts at time \( t - h \), overall uncertainty is equal to the sum of the expected squared error in the mean forecast and observed dispersion:

\[
V_{ith} = E(e_{ith}^2) + d_{ith}.
\]

(3)

Eq. (3) shows that overall earnings forecast uncertainty is comprised of two elements—a common and an idiosyncratic. One component is the variance of mean forecast errors, which is dominated by the volatility of unanticipated aggregate shocks (Lahirii and Sheng, 2010). This common element can be interpreted as uncertainty arising from future events that

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1. We should point out that BKLS Uncertainty is only partially ex post because it includes contemporaneous dispersion.
2. Barron et al. (1998) define dispersion as the sample variance of forecasts rather than the population variance and note that all of their formulas can be obtained using either definition of dispersion.
3. Under the assumption that forecasts are unbiased, the variance of mean forecast errors is the same as the expected squared error in the mean forecast.
analysts can neither predict nor have private knowledge of. The other component captures uncertainty due to differences in analysts’ information sets or forecasting models and abilities. Combined, the variance of mean forecast errors and dispersion among analysts capture the two sources of earnings forecast uncertainty faced by financial analysts. It should be noted that the theoretical construct of earnings forecast uncertainty expressed in Eq. (3) is inherently ex ante because it is based on information at time $t-h$. Therefore, it should be constructed using information known to analysts at the time they make their forecasts and should not be affected by future information, such as actual earnings.

The rest of this section reviews two popular measures of uncertainty used in the literature, namely, forecast dispersion and BKLS Uncertainty. Then we suggest a third measure that, we argue, is better aligned with the theoretical construct of uncertainty than the other two.

2.1. Forecast dispersion

Due to the ready availability of point forecasts, dispersion among analysts, as defined in Eq. (1), has been widely used in the accounting and finance literature as a proxy for uncertainty about future earnings (see, e.g. Imhoff and Lobo, 1992; Barron and Stuerke, 1998; Zhang, 2006a, 2006b; Yeung, 2009). The use of dispersion is attractive because it provides an ex ante measure of uncertainty. However, Abarbanell et al. (1995) and Johnson (2004), among others, point out that dispersion does not capture uncertainty fully. Therefore, it is questionable whether dispersion alone is a reliable measure of uncertainty in all settings.

As shown in Eq. (3), dispersion is only one part of overall forecast uncertainty and their difference is the variance of mean forecast errors, which is dominated by the variance of unanticipated shocks that accumulate over horizons, i.e. the longer the period over which unexpected events may occur, the larger the difference on average. It also suggests that the robustness of the relation between dispersion and uncertainty depends on the variability of aggregate shocks over time. In relatively stable time periods where the variability of aggregate shocks is low, dispersion will be a good proxy for unobservable uncertainty. In periods where the volatility of aggregate shocks is high, dispersion can become a tenuous proxy for uncertainty. Thus, our analysis helps the understanding of the appropriateness of dispersion as a proxy for forecast uncertainty.

Prior studies, such as L’Her and Suret (1996), interpret changes in dispersion to imply changes in uncertainty. However, the recent research by Barron et al. (2009) reveals that levels of dispersion reflect levels of uncertainty and changes in dispersion measure changes in information asymmetry. Based on this argument, the authors are able to reconcile seemingly conflicting evidence in the accounting and finance literature. Therefore, researchers and investors should be especially cautious when using dispersion to measure changes in uncertainty.

2.2. BKLS uncertainty

An alternative measure of uncertainty has been proposed by Barron et al. (1998). The authors suggest that one could use the actual squared error in the mean forecast as a proxy for the expected squared error. This measure has been extensively used to study analysts’ information environment revealed by their earnings forecasts (see, e.g. Barron et al., 2002; Liang, 2003; Botosan and Stanford, 2005; Doukas et al., 2006; Yeung, 2009). Empirically, they estimate earnings forecast uncertainty using the following equation:

$$V_{tb} = e_{tb}^2 + d_{tb}. \quad (4)$$

Since forecast errors are known to analysts only after the announcement of actual earnings, Eq. (4) yields an ex post measure of uncertainty. Hence, the reliability of BKLS Uncertainty as a proxy for ex ante uncertainty at the time the forecasts are made is an empirical issue.

Using a unique data set in economics where professional forecasters provide density forecasts for output growth and inflation, Lahiri and Sheng (2010) compare BKLS Uncertainty to a self-reported measure of uncertainty directly.\(^4\) They find that, compared to the survey measure of uncertainty, which is widely accepted to be the best available estimate of subjective uncertainty, ex post uncertainty based on Eq. (4) is considerably more volatile. This is because forecast errors are ex post quantities, which should not affect analysts’ uncertainty at the time they issue a forecast.\(^5\)

To further illustrate this point, consider AMR Corporation whose principal subsidiary is American Airlines. Following the 9/11 attack, American Airlines suffered a tremendous loss in revenues due to a two-day flying suspension, a drop in the number of passengers and ticket prices and the loss of two planes with passengers and employees. This enormous drop in earnings in the last two quarters of the fiscal year results in a large error in the mean forecast, which, in turn, increases BKLS Uncertainty tremendously. However, analysts unlikely become overly uncertain prior to the occurrence of such an

\(^4\) The data are taken from Survey of Professional Forecasters (SPF) that is provided by the Federal Reserve Bank of Philadelphia. Besides their point forecasts, forecasters are also asked to provide their probability distributions for output growth and inflation. See Lahiri and Sheng (2010) for the details of the data set.

\(^5\) In this paper we focus on the conditional relation between forecast errors and forecast uncertainty, that is, conditional on the information available to analysts at the time they issue a forecast. The unconditional relation implies that forecast errors should reflect forecast uncertainty in history and on average. However, the unconditional relation is unlikely to be useful in practical applications.
extreme event. A look at the data for the year 2001 for AMR Corporation reveals that forecast uncertainty estimated by the BKLS measure at three quarters ahead increased almost 350 times from the year before, while it increased less than five times based on dispersion or the new suggested measure, as discussed in the next section.6

Finally, BKLS Uncertainty relies on an assumption that is unlikely to hold in practice. Namely, actual earnings are assumed to be exogenous and hence, unaffected by analysts’ forecasts. However, there is a large number of studies showing that managers manipulate earnings to meet or beat analysts’ forecasts (see, e.g. Degeorge et al., 1999; Abarbanell and Lehavy, 2003). To the extent that earnings management exists, common uncertainty measured by the actual squared error in the mean forecast will be understated.

2.3. New uncertainty measure

Since uncertainty is essentially an ex ante concept attached to a forecast before the actual earnings are known, it must be constructed using data available to analysts at the time forecasts are made. Eq. (3) shows that earnings forecast uncertainty is comprised of dispersion, $d_{th}$, and the variance of mean forecast errors, which we denote by $\sigma^2_{th}$. The intuition behind this uncertainty measure is as follows. Uncertainty arises from two sources: the error components in common information and those in private information. The $\sigma^2_{th}$ captures primarily the imprecision of common information, and $d_{th}$ reflects the imprecision in analysts’ idiosyncratic information and diversity in their forecasting models.7

Estimating the variance of mean forecast errors empirically poses a problem because some periods are likely to be more volatile than others and volatile periods tend to cluster. To deal with these problems, Engle (1982) develops the celebrated Autoregressive Conditional Heteroskedasticity (ARCH) model, which is generalized by Bollerslev (1986) to form GARCH. These models can be used to estimate volatility conditional on historical data and are therefore suitable for our purpose. The technique is now a standard approach for modeling different types of uncertainty in economics and finance.8 In our setting, the GARCH model assumes that the volatility of mean forecast errors depends on past forecast errors and lagged earnings forecast uncertainty. Specifically, the method uses historical mean forecast errors to provide an estimate of the variance of mean forecast errors for the current period. We estimate a simple GARCH (1,1) model and generate the conditional variance, $\sigma^2_{th}$. Then our uncertainty measure is the sum of the projected variance of mean forecast errors and the observed dispersion:

$$\hat{V}_{th} = \sigma^2_{th} + d_{th}. \quad (5)$$

This procedure provides estimated overall uncertainty that can be used in settings where other measures cannot, such as when firm’s operations are affected by unanticipated events, bankruptcy and large restructuring charges and when the construct of interest is the change in uncertainty. The new measure is easy to implement, since many widely used statistical software packages, such as Stata, Eviews and SAS, contain pre-programmed routines for the estimation of the GARCH models. In Appendix A, we provide theoretical and implementation details on the GARCH model estimation.

3. Empirical analysis

To illustrate the performance of three alternative uncertainty measures, we test the hypothesis, assuming that the Bayesian model is descriptive, that analysts rely more on reported earnings to revise their forecasts when the ex ante uncertainty in future earnings is relatively high. In a recent paper, Yeung (2009) gives a detailed discussion and conducts a regression test of this hypothesis. In this section, we extend Yeung’s analysis by examining the relation between forecast revisions and earnings surprises conditional on analysts’ uncertainty for different forecast horizons.

The initial sample includes all US firms in the I/B/E/S Detail tape for the 24-year period 1984–2007. Following Yeung (2009), we require that forecasts are made within 90 days before the earnings announcement and revised forecasts are issued no later than 30 days after the earnings announcement. If an analyst makes more than one forecast in the period before or after the announcement, we include only the forecasts that are closest to the earnings announcement date. To obtain a better measure of dispersion, we require that there are at least three forecasts for each firm/year in the 90 days before the earnings announcement.

Fig. 1 provides a general timeline of events related to our empirical analysis. We split the data based on the length of the forecast horizon and examine forecasts of annual earnings that analysts issue before and after three prior quarterly earnings announcements. Horizons 1, 2 and 3 forecasts are made around earnings announcements of third, second and first quarter earnings for the current year, respectively. For example, the forecasts at Horizon 1 are issued from 90 days before the third quarter’s earnings announcement to 30 days after. We examine analysts’ forecast revision behavior on a horizon-by-horizon basis, since analysts’ information and uncertainty varies over horizons (Barron et al., 2002). As the forecast horizon becomes longer, the variance of mean forecast errors tends to become the dominant component of uncertainty, and by construction, the new measure is expected to provide a better estimate of uncertainty under this situation.

6 Uncertainty at three quarters ahead is calculated using forecasts issued during the months of February, March and April and AMR Corporation has a December 31 fiscal year-end.
7 As in Barron et al. (1998), $\sigma^2_{th}$ (SE in their model), captures common uncertainty and a fraction of idiosyncratic uncertainty. As the number of analysts following a given firm increases, idiosyncratic errors will be averaged out in the mean forecast and $\sigma^2_{th}$ will reflect only common uncertainty.
8 See Bollerslev et al. (1992) for an early survey of its applications and Andersen et al. (2010) for more recent advances.
In our analysis we require that sample firms have no missing forecasts and actual values, have data available for all 24 years and are listed on Compustat during the sample period. This leaves us with 128 firms for Horizon 1, 107 for Horizon 2 and 108 for Horizon 3. Table 1 provides information about the industry distribution of our sample firms. There are a total of 166 unique firms for all three horizons, with Durable manufacturers accounting for 25 percent of the sample, followed by Utilities accounting for 10 percent.

We run the following regression to examine the relation between forecast revision and earnings surprise conditional on analysts’ uncertainty for different horizons:

\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Surprise} \times \text{Uncertainty} + \beta_3 \times \text{Uncertainty} + \beta_4 \times \text{Surprise} \times \text{Neg} + \beta_5 \times \text{Neg} + \epsilon \]  

Fig. 1. Timeline. Notes: This figure describes the timeline of events assumed in our empirical analysis. Analysts are making forecasts for annual earnings, which are realized at the target fiscal year-end. Horizons 1, 2 and 3 forecasts are made around earnings announcements one, two and three quarters prior to the end of the target year, respectively. Uncertainty is measured using forecasts issued in the 90 days before the current quarter’s earnings announcement (EA). Revised forecasts are issued in the 30 days following the earnings announcement.

Table 1
Sample firms industry distribution.

<table>
<thead>
<tr>
<th>Industry</th>
<th>Number</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mining and construction</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Food</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Textiles and printing/publishing</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Chemicals</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Pharmaceuticals</td>
<td>9</td>
<td>5</td>
</tr>
<tr>
<td>Extractive industries</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>Durable manufacturers</td>
<td>42</td>
<td>25</td>
</tr>
<tr>
<td>Computers</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Transportation</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Utilities</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Retail</td>
<td>8</td>
<td>5</td>
</tr>
<tr>
<td>Financial institutions</td>
<td>12</td>
<td>7</td>
</tr>
<tr>
<td>Insurance and real estate</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Services</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Other industry</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>166</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Notes: Sample firms are taken from I/B/E/S Detail tapes, without missing forecasts and actual values for the 24-year period 1984–2007, when earnings announcement dates fall within 90 days following the quarter end. In our analysis, data are split based on the period when the forecasts are issued relative to the end of the target year. We refer to these time periods as Horizon 1, Horizon 2 and Horizon 3. The Horizon 1 forecasts are issued from 90 days before the last quarter’s earnings announcement to 30 days after. Horizon 2 and Horizon 3 forecasts are made around earnings announcements two and three quarters before the end of the target year, respectively. More details are provided in Fig. 1. The final sample includes 128 firms at Horizon 1, 107 firms at Horizon 2 and 108 firms at Horizon 3. This table includes the unique sample of firms for all horizons.

The industry classification is as in Barth et al. (1999): Mining and Construction SIC codes 1000–1999, excluding 1300–1399; Food 2000–2111; Textiles and printing/publishing 2200–2780; Chemicals 2800–2824, 2840–2899; Pharmaceuticals 2830–2836; Extractive industries 2900–2999, 1300–1399; Durable manufacturers 3000–3999, excluding 3570–3579 and 3670–3679; Computers 7370–7379, 3570–3579, 3670–3679; Transportation 4000–4899; Retail 5000–5999; Financial institutions 6000–6411; Insurance and real estate 6500–6999; Services 7000–8999, excluding 7370–7379; Other industry 700 (Agricultural services).

In our analysis we require that sample firms have no missing forecasts and actual values, have data available for all 24 years and are listed on Compustat during the sample period. This leaves us with 128 firms for Horizon 1, 107 for Horizon 2 and 108 for Horizon 3. Table 1 provides information about the industry distribution of our sample firms. There are a total of 166 unique firms for all three horizons, with Durable manufacturers accounting for 25 percent of the sample, followed by Utilities accounting for 10 percent.

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\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Surprise} \times \text{Uncertainty} + \beta_3 \times \text{Uncertainty} + \beta_4 \times \text{Surprise} \times \text{Neg} + \beta_5 \times \text{Neg} + \epsilon \]  

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9. This requirement restricts the sample to large and heavily followed firms and hence, in our sample the variance of mean forecast errors is likely to approximate the variance of unanticipated aggregate shocks.

10. For convenience, firm, year, and horizon subscripts are omitted.
where

Revision is defined as the revised mean annual forecast minus the mean annual forecast prior to the earnings announcement;

Surprise is defined as I/B/E/S actual earnings for the current quarter minus the mean forecast for the current quarter;

Uncertainty represents the natural logarithm of one of the following three uncertainty measures;

Dispersion is the forecast dispersion defined in Eq. (1), measured before the current quarter’s earnings announcement;

BKLS Uncertainty is the uncertainty measure proposed by Barron et al. (1998), defined in Eq. (4) as the sum of the squared error in the mean forecast and dispersion, measured before the current quarter’s earnings announcement;

ST Uncertainty is the new uncertainty measure defined in Eq. (5), calculated before the current quarter’s earnings announcement.

Neg is equal to 1 if the reported earnings are negative and zero otherwise.11

We also include year and firm fixed effects in our regressions. Uncertainty variables are measured using forecasts issued in the 90 days before the respective earnings announcement. Table 2 presents descriptive statistics for the variables used in our analysis. Revision has a negative mean in Horizons 1 and 2 and a positive mean in Horizon 3, implying that analysts are more likely to revise their forecasts upwards at longer horizons and downwards as the end of the target year is approaching. When it comes to the uncertainty measures, several interesting observations emerge. First, Dispersion is always lower than the other two, suggesting that this proxy likely understates the level of uncertainty, consistent with it capturing only one component of uncertainty. This is especially true at longer horizons. The medians show the same

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**Table 2**

Descriptive statistics.

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>Mean</th>
<th>Std.</th>
<th>Q1</th>
<th>Median</th>
<th>Q3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Horizon 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revision</td>
<td>3001</td>
<td>-0.015</td>
<td>0.195</td>
<td>-0.033</td>
<td>0.000</td>
<td>0.028</td>
</tr>
<tr>
<td>Surprise</td>
<td>2983</td>
<td>-0.004</td>
<td>0.166</td>
<td>-0.014</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Uncertainty</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
<td>3072</td>
<td>0.025</td>
<td>0.171</td>
<td>0.000</td>
<td>0.001</td>
<td>0.006</td>
</tr>
<tr>
<td>BKLS Uncertainty</td>
<td>3072</td>
<td>0.232</td>
<td>5.295</td>
<td>0.001</td>
<td>0.004</td>
<td>0.026</td>
</tr>
<tr>
<td>ST Uncertainty</td>
<td>3072</td>
<td>0.154</td>
<td>1.016</td>
<td>0.003</td>
<td>0.012</td>
<td>0.062</td>
</tr>
<tr>
<td><strong>Panel B: Horizon 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Revision</td>
<td>2469</td>
<td>-0.008</td>
<td>0.279</td>
<td>-0.040</td>
<td>0.003</td>
<td>0.044</td>
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<tr>
<td>Surprise</td>
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<td>0.066</td>
<td>0.252</td>
<td>-0.011</td>
<td>0.005</td>
<td>0.028</td>
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<td>Uncertainty</td>
<td></td>
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<td></td>
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<tr>
<td>Dispersion</td>
<td>2568</td>
<td>0.071</td>
<td>0.826</td>
<td>0.000</td>
<td>0.003</td>
<td>0.014</td>
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<tr>
<td>BKLS Uncertainty</td>
<td>2568</td>
<td>0.864</td>
<td>13.185</td>
<td>0.002</td>
<td>0.014</td>
<td>0.089</td>
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<td>ST Uncertainty</td>
<td>2568</td>
<td>0.764</td>
<td>14.487</td>
<td>0.007</td>
<td>0.036</td>
<td>0.193</td>
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<tr>
<td><strong>Panel C: Horizon 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Revision</td>
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<td>0.263</td>
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<td>0.048</td>
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<td>Surprise</td>
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<td>0.010</td>
<td>0.126</td>
<td>-0.009</td>
<td>0.005</td>
<td>0.030</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
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<td>0.449</td>
<td>0.001</td>
<td>0.003</td>
<td>0.014</td>
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<td>BKLS Uncertainty</td>
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<td>24.146</td>
<td>0.003</td>
<td>0.020</td>
<td>0.117</td>
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<tr>
<td>ST Uncertainty</td>
<td>2592</td>
<td>0.960</td>
<td>6.115</td>
<td>0.010</td>
<td>0.053</td>
<td>0.204</td>
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</tbody>
</table>

**Notes:** This table presents the descriptive statistics for variables used in the statistical analysis. Variables are defined as follows. Revision is the revised mean annual forecast minus the mean annual forecast prior to the earnings announcement; Surprise is equal to I/B/E/S actual earnings for the current quarter minus the mean forecast for the current quarter; Dispersion is the forecast dispersion defined in Eq. (1), measured before the current quarter’s earnings announcement; BKLS Uncertainty is the uncertainty measure proposed by Barron et al. (1998), defined in Eq. (4) as the sum of the squared error in the mean forecast and dispersion, measured before the current quarter’s earnings announcement; ST Uncertainty is the new uncertainty measure defined in Eq. (5), calculated before the current quarter’s earnings announcement. Further reported results use the natural logarithm transformations of the uncertainty measures.

Horizons 1, 2 and 3 are defined in Table 1 and Fig. 1.

The sample is obtained as described in Table 1.

---

11 Yeung (2009) adds Surprise and AR1 as additional control variables for the noise in earnings surprises. Since our goal is to examine the performance of three alternative measures of uncertainty, we maintain a parsimonious regression model.
Second, \textit{ST Uncertainty} has the highest median among the three measures for all horizons, while \textit{BKLS Uncertainty} has the highest mean. This is not surprising considering the fact that one of the components of \textit{BKLS Uncertainty} is the error in the mean forecast, which may result in substantially overstated uncertainty estimates in periods when significant unanticipated events occur causing a large error in the mean forecast.

To further examine their performance, we plot the medians of the uncertainty measures over time and by horizon. The results are presented in Fig. 2. Clearly, \textit{Dispersion} is always lower than the other two in all years and horizons, providing additional evidence that it likely understates the level of uncertainty especially at long horizons. Another interesting finding is the increase in uncertainty in year 2001. The moderate increase in \textit{Dispersion} and \textit{ST Uncertainty} may be attributed to an economy-wide recession that occurred from March to November 2001. However, while it rose only slightly at Horizon 1, \textit{BKLS Uncertainty} jumped a lot at Horizons 2 and 3. For example, at Horizon 3 \textit{BKLS Uncertainty} rose dramatically—an increase of almost 400 percentage points in year 2001 relative to year 2000. This is possibly due to an enormous drop in earnings for many firms in the last two quarters of the fiscal year, following the event of 9/11. This results in a large error in the mean forecast, which, in turn, increases BKLS Uncertainty. As noted earlier, however, such an extreme event could not be predicted prior to its occurrence and therefore, should not affect analysts’ uncertainty at the time they issue a forecast at Horizon 3. BKLS Uncertainty may give misleading inferences in periods when significant unanticipated events occur after the forecasts are made.\footnote{However, it should be noted that even if \textit{Dispersion} understates uncertainty, it may still be a good cross-sectional indicator of the relative level of uncertainty, which indeed has been established in the literature (see, e.g. Barron and Stuerke, 1998; Lahiri and Sheng, 2010).} \footnote{However, if such events are partially anticipated by analysts, then true uncertainty is likely to increase, which, in turn, leads to an increase in BKLS Uncertainty.} \footnote{Similar results are obtained when we plot the medians of the log-transformed and scaled uncertainty measures by firm price at the beginning of the quarter.}
Table 3
Spearman correlations.

<table>
<thead>
<tr>
<th></th>
<th>Dispersion</th>
<th>BKLS Uncertainty</th>
<th>ST Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Horizon 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dispersion</td>
<td>1.000</td>
<td>0.867</td>
<td>0.775</td>
</tr>
<tr>
<td>BKLS Uncertainty</td>
<td>0.867</td>
<td>1.000</td>
<td>0.761</td>
</tr>
<tr>
<td>ST Uncertainty</td>
<td>0.775</td>
<td>0.761</td>
<td>1.000</td>
</tr>
</tbody>
</table>

| **Panel B: Horizon 2** |            |                  |                |
| Dispersion    | 1.000      | 0.852            | 0.809          |
| BKLS Uncertainty | 0.852  | 1.000            | 0.780          |
| ST Uncertainty | 0.809    | 0.780            | 1.000          |

| **Panel C: Horizon 3** |            |                  |                |
| Dispersion    | 1.000      | 0.811            | 0.780          |
| BKLS Uncertainty | 0.811  | 1.000            | 0.741          |
| ST Uncertainty | 0.780    | 0.741            | 1.000          |

Notes: All correlations are statistically significant at the two-tailed 1 percent level. Variables are defined in Table 2.

Another observation worth mentioning from Fig. 2 is the substantial difference between ST Uncertainty and BKLS Uncertainty, especially at Horizons 2 and 3, for all years except 2001. Both ST Uncertainty and BKLS Uncertainty use mean forecast errors as inputs, which are likely to be biased towards zero due to earnings management. BKLS Uncertainty has the level of the squared error in the mean forecast as one component and hence, it is unambiguously understated, if earnings management occurs. However, ST Uncertainty may not be understated, since the constant term in the GARCH model specification removes any persistent bias in the mean forecast due to earnings management (see Eq. (A1)) and the conditional variance is estimated based on zero-mean, serial-uncorrelated mean forecast errors (see Eq. (A2)). The degree of underestimation for ST Uncertainty, however, depends on whether earnings management is always in the same direction and whether the level of earnings management is persistent over time. For example, consider an increase in actual earnings to meet the mean forecast by similar amounts for all years for a given firm. Then, the mean forecast errors will be understated over time, but the variance and thus ST Uncertainty will remain roughly the same. As another example, if an increase in actual earnings to meet the mean forecast is unevenly spread over time, then GARCH model estimation will remove the average level of earnings management bias and as a result, ST Uncertainty would only partially avoid the endogeneity problem.

Since all uncertainty measures, especially BKLS Uncertainty, display a much higher mean than median, they are positively skewed and may include outliers. To alleviate the impact of possible outliers and non-normality of the distribution of the uncertainty measures, we use the natural logarithm transformations of these variables in our regression analysis (Barron and Stuerke, 1998). Table 3 presents the Spearman correlation coefficients among the transformed uncertainty variables. The three measures are highly correlated, with correlation coefficients of 0.741 or higher for all horizons. This is not surprising, since both BKLS Uncertainty and ST Uncertainty have Dispersion as one of the components.15

We run regressions for each horizon separately to test the hypothesis that analysts rely more on reported earnings to revise their forecasts when the ex ante uncertainty in future earnings is relatively high. Since it is constructed to capture primarily the variance of aggregate shocks that accumulate over time, ST Uncertainty should perform better than the other two measures at longer horizons. As a result, we expect that the coefficient on Surprise × Uncertainty is positive and larger when ST Uncertainty is used rather than Dispersion or BKLS Uncertainty, especially at longer horizons.

The main regression results are presented in Table 4. Panel A shows the coefficient estimates on Surprise × Uncertainty and the corresponding standard errors from regressions of Revision on Surprise, Surprise × Uncertainty and Uncertainty. At Horizon 1, all three measures give the similar result that, conditional on analysts’ uncertainty, forecast revisions and earnings surprises are positively associated. As the forecast horizon gets longer, ST Uncertainty gives the highest coefficient estimates. This is especially pronounced at Horizon 3, where ST Uncertainty yields an estimate of 0.173, compared to 0.081 and 0.102 when using Dispersion and BKLS Uncertainty, respectively. Panel B reports the estimated results from the full regression model as specified in Eq. (6). At Horizon 2, the coefficient on Surprise × ST Uncertainty is estimated to be 0.172 versus 0.121 on Surprise × Dispersion and 0.110 on Surprise × BKLS Uncertainty. At Horizon 3, the estimates are 0.257 when using ST Uncertainty versus 0.092 and 0.166 when using Dispersion and BKLS Uncertainty as proxies for forecast uncertainty, respectively. Taken together, the estimated results reported in Table 4 show that the relation between Revision and

15 Similar results are also obtained when Pearson correlations are calculated.
The results of this analysis are presented in Table 5. In one regression, uncertainty is measured by

\[ \text{Dispersion} \times \text{Surprise} \]

and

\[ \text{ST Uncertainty} \]

vs.

\[ \text{BKLS Uncertainty} \]

Panel A presents the coefficient estimates and standard errors of \( \beta_2 \) from the following regression (standard errors are in parentheses):

\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Dispersion} + \beta_3 \times \text{ST Uncertainty} + \beta_4 \times \text{BKLS Uncertainty} + u \]

Panel B presents the coefficient estimates and standard errors of \( \beta_2 \) from the regression:

\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Uncertainty} + \beta_3 \times \text{ST Uncertainty} + \beta_4 \times \text{BKLS Uncertainty} + \beta_5 \times \text{Neg} + \epsilon \]

where Uncertainty is measured by Dispersion, BKLS Uncertainty and ST Uncertainty and Neg is equal to 1 if the reported earnings are negative and zero otherwise. All other variables are defined in Table 2.

Notes:
Panel A presents the coefficient estimates and standard errors of \( \beta_2 \) from the regression:
\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Dispersion} + \beta_3 \times \text{ST Uncertainty} + \beta_4 \times \text{BKLS Uncertainty} + u \]

Panel B presents the coefficient estimates and standard errors of \( \beta_2 \) from the regression:
\[ \text{Revision} = \beta_0 + \beta_1 \times \text{Surprise} + \beta_2 \times \text{Uncertainty} + \beta_3 \times \text{ST Uncertainty} + \beta_4 \times \text{BKLS Uncertainty} + \beta_5 \times \text{Neg} + \epsilon \]

while Uncertainty is measured by Dispersion, BKLS Uncertainty and ST Uncertainty and Neg is equal to 1 if the reported earnings are negative and zero otherwise. All other variables are defined in Table 2.

** Denotes significance at the 1 percent level (two-tailed test).

---

**Table 4**

Regression results by horizon.

<table>
<thead>
<tr>
<th></th>
<th>Horizon 1</th>
<th>Horizon 2</th>
<th>Horizon 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surprise × Dispersion</td>
<td>0.094**</td>
<td>0.071**</td>
<td>0.081**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Surprise × BKLS Uncertainty</td>
<td>0.078**</td>
<td>0.080**</td>
<td>0.102**</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.006)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Surprise × ST Uncertainty</td>
<td>0.090**</td>
<td>0.083**</td>
<td>0.173**</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.008)</td>
<td>(0.014)</td>
</tr>
</tbody>
</table>

---

**Table 5**

Test of statistical difference in coefficients from regressions on stacked data.

<table>
<thead>
<tr>
<th></th>
<th>Horizon 1</th>
<th>Horizon 2</th>
<th>Horizon 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>BKLS Uncertainty vs. Dispersion</td>
<td>0.006</td>
<td>−0.030**</td>
<td>0.032*</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>ST Uncertainty vs. BKLS Uncertainty</td>
<td>0.019**</td>
<td>0.030**</td>
<td>0.078**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.006)</td>
<td>(0.015)</td>
</tr>
</tbody>
</table>

---

Notes:
Table 4 presents the coefficient estimates and standard errors of \( \beta_2 \) from the following regression (standard errors are in parentheses):

\[ \text{Revision} = \delta_0 + \delta_1 \times \text{Surprise} + \delta_2 \times \text{Dispersion} + \delta_3 \times \text{ST Uncertainty} + \delta_4 \times \text{BKLS Uncertainty} + u \]

where Uncertainty is measured by Dispersion or BKLS Uncertainty (Panel A) and BKLS Uncertainty or ST Uncertainty (Panel B), and BetterMeasure is an indicator variable equal to one if Uncertainty is measured by BKLS Uncertainty (Panel A) or ST Uncertainty (Panel B) and zero, otherwise. All variables are defined in Table 2.

Notes:
This table presents the coefficient estimates and standard errors of \( \delta_3 \) from the following regression (standard errors are in parentheses):

\[ \text{Revision} = \delta_0 + \delta_1 \times \text{Surprise} + \delta_2 \times \text{Dispersion} + \delta_3 \times \text{ST Uncertainty} + \delta_4 \times \text{BKLS Uncertainty} + \delta_5 \times \text{BetterMeasure} + \delta_6 \times \text{Uncertainty} + \delta_7 \times \text{ST Uncertainty} + \delta_8 \times \text{BKLS Uncertainty} + \delta_9 \times \text{Neg} + \delta_0 \times \epsilon \]

Year and firm fixed effects are included in all regression models. (Full regression results are available upon request.)

** Denotes significance at the 1 percent level (two-tailed test).

---

Surprise, conditional on analysts’ uncertainty, is much stronger when forecast uncertainty is estimated by ST Uncertainty than by Dispersion and BKLS Uncertainty, especially when the forecast horizon lengths.16

While the results in Table 4 are supportive, they do not provide a statistical test of the superiority of the new measure over the other two. For this purpose, we run regressions on stacked data where every observation appears twice using two alternative measures of uncertainty. The results of this analysis are presented in Table 5. In one regression, uncertainty is measured by

\[ \text{Dispersion} \times \text{Surprise} \]

and

\[ \text{ST Uncertainty} \]

vs.

\[ \text{BKLS Uncertainty} \]

Notes:
It should be noted that our tests are based on the magnitude of the coefficients on the interactions between Surprise and Uncertainty, which assumes that a larger and more significant coefficient is better. However, in general, the better measure of uncertainty would produce a correct coefficient, not necessarily a larger one.

16 It should be noted that our tests are based on the magnitude of the coefficients on the interactions between Surprise and Uncertainty, which assumes that a larger and more significant coefficient is better. However, in general, the better measure of uncertainty would produce a correct coefficient, not necessarily a larger one.
measured by Dispersion and BKLS Uncertainty and in another, by BKLS Uncertainty and ST Uncertainty. We define an indicator variable, BetterMeasure, equal to one if uncertainty is measured by BKLS Uncertainty (Panel A) or ST Uncertainty (Panel B) and zero, otherwise. The coefficient on BetterMeasure × Surprise × Uncertainty provides the key information about the differences in coefficients between BKLS Uncertainty and Dispersion (Panel A), and between ST Uncertainty and BKLS Uncertainty (Panel B). In Panel A, BKLS Uncertainty shows a modest improvement over Dispersion. In Panel B, the reported positive and highly significant coefficients imply that ST Uncertainty provides stronger results than BKLS Uncertainty in support of the hypothesis tested in this paper.

Finally, we examine the economic significance of the results obtained with ST Uncertainty. Since we include the interaction term Surprise × Uncertainty, the marginal effects of the earnings surprise variable have to be interpreted conditionally on the interaction with earnings forecast uncertainty. In principle, there are two sensible ways to evaluate the marginal effects. Following Jaccard et al. (1990, p. 27), we evaluate the marginal effects at the low as well as the high level of the interacted variable, i.e. earnings forecast uncertainty. Using this method we are able to examine the influences of earnings surprise on forecast revision when earnings uncertainty is high and low. In addition, we also evaluate the marginal effects at the average level of earnings forecast uncertainty.

Table 6 reports the estimated marginal effects when forecast uncertainty is constructed according to the new measure. In Panel A, at the average level of earnings forecast uncertainty, an increase in the earnings surprise by one percent raises forecast revision by 0.548 percent at Horizon 1 and by about 0.330 percent at Horizons 2 and 3. Moreover, earnings surprise has the most significant effect on forecast revision when earnings uncertainty is high for all horizons. For example, at Horizon 1, a one percent increase in Surprise is associated with a 0.754 percent increase in Revision when earnings forecast uncertainty is high and a 0.342 percent increase when uncertainty is low. The similar marginal effects are also observed in Panel B with Neg as an additional variable to control for the noise in earnings surprises. By including different forecast horizons, the results reported in Table 6 confirm and extend earlier findings in Yeung (2009).

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17 The full regression models and additional details of this analysis are presented in Table 5.
18 Our results are robust to alternative GARCH model specifications, controlling for the optimistic bias in analysts’ forecasts and scaling all continuous variables by total assets. However, the paper does not test whether changes in ST Uncertainty are superior to changes in BKLS Uncertainty.
It should be noted that ST Uncertainty has some potential drawbacks. First, it assumes that the past is a good guide to the future. Although this assumption, in one form or another, underlies all statistical analyses, there is always a risk that structural changes to earnings may alter their inherent predictability, thereby reducing the relevance of past mean forecast errors for the prediction of future errors. One should be alert to evidence of structural change and other factors that may alter the predictability of earnings and apply the new measure with caution in such situations. For example, previous research finds evidence that forecast accuracy may be affected by certain events, such as a change in accounting method (Elliott and Philbrick, 1990) or a switch to International Accounting Standards (Ashbaugh and Pincus, 2001). ST Uncertainty may not be the preferable measure of uncertainty in these settings. Second, the GARCH model estimation requires a long time-series of data without missing values, restricting its applicability to heavily followed firms. However, recent advances in the area of econometric research have proposed ways of dealing with missing data and we leave it to future research to apply our technique to the wider population of firms.

4. Conclusion

Based on the theoretical finding that uncertainty includes a common and an idiosyncratic component, we propose a new measure of earnings forecast uncertainty as the sum of the projected variance of mean forecast errors from a suitably specified GARCH model and dispersion among analysts. Compared to the existing proxies for uncertainty in the literature, the proposed measure has some obvious advantages: (i) it provides a measure of uncertainty that is conditional on observing forecasts and hence, can be estimated in practice; (ii) it is based on information available to analysts at the time they make their forecasts and gives an ex ante measure of uncertainty, thus alleviating some of the limitations associated with BKLS Uncertainty; and (iii) it captures the uncertainty arising from both analysts’ common and private information and therefore avoids the problems of using dispersion alone as a proxy for uncertainty.

We test the hypothesis that analysts rely more on reported earnings to revise their forecasts when the ex ante uncertainty in future earnings is relatively high. Our estimation results show that the positive relation between forecast revisions and earnings surprises becomes stronger at longer forecast horizons, when uncertainty is estimated by the ST measure rather than by dispersion or the BKLS measure. The main advantage of ST Uncertainty over BKLS Uncertainty comes from the way in which the expected squared error in the mean forecast is estimated. Thus, besides its three merits mentioned above, the new measure has a fourth advantage. By construction, it captures the variance of aggregate shocks that accumulate over forecast horizons and thus gives a better proxy for uncertainty especially at longer horizons. For these reasons, we recommend the use of the new measure of earnings forecast uncertainty in practice.

One can use the measure proposed here to explore to what extent analysts’ uncertainty about future earnings reflects the uncertainty due to common information versus private information. Another worthwhile extension is to estimate the expected squared error in the mean forecast by using the technique suggested in this paper and develop empirical proxies for other theoretical constructs, such as consensus and precision of information in earnings forecasts.

Appendix A. Estimation of the conditional variance $\sigma_{th}^2$ by GARCH models

A.1. GARCH (1,1) model specification

The simple GARCH (1,1) model specification at horizon $h$ is

$$
\epsilon_t = \phi_0 + \phi_1 \epsilon_{t-1} + \cdots + \phi_{h-1} \epsilon_{t-(h-1)} + \epsilon_t, \quad \epsilon_t \mid \Psi_{t-1} \sim N(0, \sigma_t^2),
$$

(A1)

$$
\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \alpha_2 \sigma_{t-1}^2.
$$

(A2)

The mean equation in (A1) is written as a function of a constant, moving average terms, and an error term. Since $\sigma_t^2$ is the one-period ahead variance equation based on past information set $\Psi_{t-1}$, it is called the conditional variance. The conditional variance equation specified in (A2) is a function of three terms: (i) a constant term, $\alpha_0$; (ii) news about volatility from the previous period, measured as the lag of the squared residual from the mean equation, $\epsilon_{t-1}^2$ (the ARCH term); and (iii) the last period’s forecast variance, $\sigma_{t-1}^2$ (the GARCH term).

A.2. Steps in estimating the conditional variance:

1. For each firm, year and horizon, calculate the mean forecast error across analysts’ earnings forecasts, $e_{th}$.
2. Remove the possible bias and autocorrelation in the mean forecast error, $e_{th}$, by fitting moving average (MA) models of varying order as in Eq. (A1). Theoretically, optimal forecasts $h$ steps ahead have dependence of order $h-1$. Hence at Horizon 1 one should fit a MA(0), at Horizon 2—a MA(1) and at Horizon 3—a MA(2) model.
3. Estimate the GARCH (1,1) model.
4. Generate the conditional variance $\hat{\sigma}_{th}^2$ using estimated model parameters.

---

19 Firm and horizon subscripts are omitted.
A.3. Some implementation details

The pre-programmed routines for the estimation of GARCH models are contained in most software, including Stata, Eviews and SAS. Very often, Eqs. (A1) and (A2) are estimated at the same time in the program such that steps 2 and 3 can be combined. For example, in Stata this is done by using the following command:

\[
\text{arch error, arch(1) garch(1) ma(h−1) [options]}
\]

where error is the mean forecast error, calculated in step 1.

Here we briefly discuss some implementation details involved in the model estimation.

(1) **Distributional assumptions**: We estimate GARCH (1,1) model by the method of maximum likelihood under the assumption that the errors are conditional normally distributed. Other distribution assumptions, such as \(t\)-distribution and the generalized error distribution, are also worth trying.

(2) **Number of ARCH and GARCH terms**: We use one ARCH and one GARCH term, i.e. GARCH (1,1) to maintain a parsimonious model that entails fewer coefficient restrictions. Since all coefficients in (A2) must be nonnegative and \(a_1 + a_2 < 1\) to ensure that the conditional variance is nonnegative and stationary, most empirical implementations of GARCH models adopt low orders for the lag lengths \(p\) and \(q\) (Bollerslev et al., 1992). Formally, one way to determine the order of \(p\) and \(q\) is to use model selection criteria, such as Aikake Information Criterion (AIC) and Schwartz Bayesian Criterion (SBC) (see, e.g. Enders, 2010, Chapter 3). The rule of thumb is to minimize the AIC or SBC, which are obtained during the estimation of alternative GARCH models. Specifically, after estimating the candidate models with different orders of \(p\) and \(q\), the model that yields the smallest AIC or SBC is selected.

(3) **Initial variance \(\sigma_0^2\)**: In our analysis we set the initial variance \(\sigma_0^2\) using the unconditional variance in the sample.\(^{20}\)

(4) **Iterative estimation control**: We use the iterative algorithm Marquardt in our estimation. As is well known, the likelihood functions of GARCH models are not always well-behaved so that convergence may not be achieved with the default setting in the particular software. One can select other iterative algorithm, such as BHHH, increase the maximum number of iterations or adjust the convergence criterion.

(5) **Data requirement**: GARCH model estimation requires a long time series of data without missing observations. If not, appropriate assumptions have to be made regarding the missing values.

References


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20 As a robustness check, we also set \(\sigma_0^2\) using the backcasting procedure. The estimation results are similar.